

Reviewer Appendix

This appendix contains proofs that we have omitted from the paper for brevity, since they pertain to results that are not central to the paper's argument. We will provide this appendix to readers upon request.

Proof that welfare at the government solution exceeds welfare at the industry solution for all $q < \hat{q}$: Let $W^g(B, q)$ denote welfare at the government optimum $(\theta^g(q), n^g(q))$ for given B and q , and let $W^i(B)$ denote welfare at the industry optimum $(\theta^s(0), \tilde{n})$. Also, let \underline{q} denote the detection probability defined implicitly by

$$f(\theta) - \alpha\theta - c(\tilde{n}) - x(q, \delta)\alpha\theta = 0$$

$$f'(\theta) - \alpha - x(q, \delta)\alpha = 0.$$

It is the probability at which the $\hat{n}(\theta, q)$ boundary shrinks to a single point, so that no club is feasible at any lower q . We are to show that $W^g(B, q) > W^i(B)$ for all $q \in [\underline{q}, \hat{q}]$. It is expedient to show this separately for intervals $[q, \tilde{q})$, $[\tilde{q}, q^s(0))$, and $[q^s(0), \hat{q}]$, with boundaries \tilde{q} and $q^s(0)$ defined below.

Note first that the Lagrangian associated with the government's optimization problem is

$$\mathcal{L}^g = n[f(\theta) - \alpha\theta - c(n)] + Bg(n\theta) + \lambda^g[f(\theta) - \alpha\theta - c(n) - x(q, \delta)\alpha\theta], \quad (\text{A1})$$

with first-order conditions

$$\frac{\partial \mathcal{L}^g}{\partial \theta} = n[f'(\theta) - \alpha] + Bg'(n\theta)n + \lambda^g[f'(\theta) - \alpha - x(q, \delta)\alpha] = 0, \quad (\text{A2})$$

$$\frac{\partial \mathcal{L}^g}{\partial n} = f(\theta) - \alpha\theta - c(n) - nc'(n) + Bg'(n\theta)\theta + \lambda^g[-c'(n)] = 0, \quad (\text{A3})$$

$$\frac{\partial \mathcal{L}^g}{\partial \lambda^g} = f(\theta) - \alpha\theta - c(n) - x(q, \delta)\alpha\theta = 0. \quad (\text{A4})$$

Solving (A2) for λ^g gives

$$\lambda^g = \frac{n[f'(\theta) - \alpha] + Bg'(n\theta)n}{-[f'(\theta) - \alpha - x(q, \delta)\alpha]}. \quad (\text{A5})$$

Moreover, the three first-order conditions can be manipulated to yield

$$f'(\theta)\theta - \alpha\theta - x(q, \delta)\alpha\theta + nc'(n) = 0,$$

or, rearranging,

$$f'(\theta) - \alpha - x(q, \delta)\alpha = -\frac{nc'(n)}{\theta} < 0,$$

where the inequality uses that only solutions with $n \geq \tilde{n}$ are stable, and that (A3) fails at $n = \tilde{n}$. It follows that the denominator of (A5) is positive.

Consider now $q^s(0)$, which is defined implicitly by

$$f(\theta^s(0)) - \alpha\theta^s(0) - c(n^s(0)) - x(q, \delta)\alpha\theta^s(0) = 0.$$

That is, $q^s(0)$ is the detection probability such that, if $B = 0$, the social optimum $(\theta^s(0), n^s(0))$ satisfies the no-cheating constraint with equality, and is therefore both feasible and optimal for the government. By revealed preference, then, $W^g(0, q^s(0)) > W^i(0)$, whereas by definition of \hat{q} , $W^g(0, \hat{q}) = W^i(0)$. Refer back now to (A5) and note that the bracketed term in the numerator is zero at $\theta^s(0) = \theta^g(q^s(0))$. Since $\theta^g(q)$ is increasing in q , the term becomes negative for all $q > q^s(0)$, requiring λ^g to become negative as well. Using this, we have by the envelope theorem that

$$\frac{\partial W^g(0, q)}{\partial q} = \frac{\partial \mathcal{L}^g}{\partial q} = -\lambda^g x_q(q, \delta) \alpha \theta^g(q) < 0. \quad (\text{A6})$$

It follows that $W^g(0, q) > W^i(0)$ for all $q \in [q^s(0), \hat{q}]$. But since $n^g(q) \geq n^s(0) > \tilde{n}$ and $\theta^g(q) \geq \theta^s(0)$, we also have by the envelope theorem that

$$\frac{\partial W^g(B, q)}{\partial B} = \frac{\partial \mathcal{L}^g}{\partial B} = Ng(n^g(q) \theta^g(q)) > Ng(\tilde{n} \theta^s(0)) = \frac{\partial W^i(B)}{\partial B}.$$

It follows that $W^g(B, q) > W^i(B)$ for all $q \in [q^s(0), \hat{q}]$ as well.

Consider next $\tilde{q} < q^s(0)$, which is defined implicitly by

$$f(\theta^s(0)) - \alpha \theta^s(0) - c(\tilde{n}) - x(q, \delta) \alpha \theta^s(0) = 0.$$

That is, \tilde{q} is the detection probability at which the unconstrained industry optimum $(\theta^s(0), \tilde{n})$ satisfies the no-cheating constraint with equality, and is therefore feasible for the government as well. Substituting $(\theta^s(0), \tilde{n})$ into (A3) yields, however, that $\partial \mathcal{L}^g / \partial n > 0$, implying that this combination is not optimal for the government. By revealed preference, therefore, $W^g(B, \tilde{q}) > W^i(B)$. Moreover, since $\theta^g(q) < \theta^s(0)$ for $q < q^s(0)$, the bracketed term in the numerator of (A5) becomes positive, implying that λ^g must be positive for all values of B . As a result, equation (A6) now changes sign, implying that $W^g(B, q) > W^i(B)$ for all $q \in [\tilde{q}, q^s(0)]$.

Lastly, for any $q \in [\underline{q}, \tilde{q}]$, the no-cheating constraint binds on the industry club. To see this, note that the Lagrangian associated with the industry club's optimization problem is

$$\mathcal{L}^i = f(\theta) - \alpha \theta - c(n) + \lambda^i [f(\theta) - \alpha \theta - c(n) - x(q, \delta) \alpha \theta],$$

with Kuhn-Tucker conditions

$$\frac{\partial \mathcal{L}^i}{\partial \theta} = f'(\theta) - \alpha + \lambda^i [f'(\theta) - \alpha - x(q, \delta) \alpha] = 0, \quad (\text{A7})$$

$$\frac{\partial \mathcal{L}^i}{\partial n} = -c'(n) + \lambda^i [-c'(n)] = 0, \quad (\text{A8})$$

$$\frac{\partial \mathcal{L}^i}{\partial \lambda^i} = f(\theta) - \alpha \theta - c(n) - x(q, \delta) \alpha \theta \geq 0, \quad (\text{A9})$$

$$\lambda^i \frac{\partial \mathcal{L}^i}{\partial \lambda^i} = 0, \quad \lambda^i \geq 0.$$

Conditions (A7) and (A8) imply that whenever the constraint binds (so $\lambda^i > 0$), the optimal club size remains $n^i = \tilde{n}$, and the term in brackets in (A7) must be negative. Given this, totally differentiating the remaining conditions w.r.t. λ^i , θ , and q gives

$$\begin{vmatrix} f' - \alpha - x\alpha & (1 + \lambda^i)f'' \\ 0 & f' - \alpha - x\alpha \end{vmatrix} \begin{vmatrix} d\lambda^i \\ d\theta \end{vmatrix} = \begin{vmatrix} \lambda^i x_q \alpha \\ x_q \alpha \theta \end{vmatrix} dq.$$

By Cramer's rule, we therefore have

$$\frac{d\lambda^i}{dq} = \frac{\begin{vmatrix} \lambda^i x_q \alpha & (1 + \lambda^i)f'' \\ x_q \alpha \theta & f' - \alpha - x\alpha \end{vmatrix}}{\begin{vmatrix} f' - \alpha - x\alpha & (1 + \lambda^i)f'' \\ 0 & f' - \alpha - x\alpha \end{vmatrix}} = \frac{\overbrace{\lambda x_q \alpha}^{(-)} \overbrace{(f' - \alpha - x\alpha)}^{(-)} - \overbrace{x_q \alpha \theta}^{(-)} \overbrace{(1 + \lambda^i)f''}^{(-)}}{\underbrace{(f' - \alpha - x\alpha)^2}_{(+)}} \geq 0, \quad (\text{A10})$$

$$\frac{d\theta^i}{dq} = \frac{\begin{vmatrix} f' - \alpha - x\alpha & \lambda^i x_q \alpha \\ 0 & x_q \alpha \theta \end{vmatrix}}{\begin{vmatrix} f' - \alpha - x\alpha & (1 + \lambda^i)f'' \\ 0 & f' - \alpha - x\alpha \end{vmatrix}} = \frac{\overbrace{-x_q \alpha \theta}^{(+)}}{\underbrace{(f' - \alpha - x\alpha)^2}_{(+)}} > 0. \quad (\text{A11})$$

Evaluated at \tilde{q} , where $\lambda^i = 0$, both derivatives are negative, implying that when q drops marginally below \tilde{q} , λ^i becomes positive, i.e., the constraint becomes binding, and θ^i drops below $\theta^s(0)$. Moreover, (A11) implies that further reductions in q further reduce θ^i , which then by condition (A7) implies that λ^i stays positive.

That the no-cheating constraint binds at all industry optima when $q \in [\underline{q}, \tilde{q}]$ implies again that these optima are feasible for the government as well. But since again $\partial \mathcal{L}^g / \partial n > 0$ at these industry optima, we again have that $W^g(B, q) > W^i(B)$ for all q in this range. Combining this with our earlier results for ranges $[\tilde{q}, q^s(0))$ and $[q^s(0), \hat{q}]$, we therefore find that $W^g(B, q) > W^i(B)$ for all $q \in [\underline{q}, \hat{q}]$. \square

Comparative statics for the industry club solution with consumer heterogeneity: Introduce a new variable $h \geq 0$ to parameterize consumer heterogeneity, and add it as an argument to the $\gamma(n)$ function defined in the text. At the original industry equilibrium with homogeneous consumers ($h = 0$), the club size is \tilde{n} , and $\gamma(\tilde{n}, 0) = 1$. Consider now an increase in consumer heterogeneity that ‘‘pivots’’ the $\gamma(n, h)$ function clockwise around $\gamma(\tilde{n}, 0)$. Then

$$\gamma_n(n, h) \begin{cases} = 0 & \text{for all } n, h = 0 \\ < 0 & \text{for all } n, h > 0 \end{cases}$$

$$\gamma_h(n, h) \begin{cases} > 0 & \text{for } n < \check{n}, h \geq 0 \\ = 0 & \text{for } n = \check{n}, h \geq 0 \\ < 0 & \text{for } n > \check{n}, h \geq 0 \end{cases}$$

and

$$\gamma_{nh}(n, h) < 0 \quad \text{for all } n, h \geq 0.$$

The industry club's optimization problem becomes

$$\max_{\theta, n} I = \gamma(n, h)f(\theta) - \alpha\theta - c(n),$$

since the club by assumption charges all consumers the same price, which must then equal the marginal consumer's willingness to pay, $\gamma(n, h)f(\theta)$. The first-order conditions are

$$\frac{\partial I}{\partial \theta} = \gamma(n, h)f'(\theta) - \alpha = 0, \quad (\text{A12})$$

$$\frac{\partial I}{\partial n} = \gamma_n(n, h)f(\theta) - c'(n) = 0. \quad (\text{A13})$$

Totally differentiating w.r.t. θ , n , and h gives

$$\begin{bmatrix} \gamma f'' & \gamma_n f' \\ \gamma_n f' & \gamma_{nn} f - c'' \end{bmatrix} \begin{bmatrix} d\theta \\ dn \end{bmatrix} = \begin{bmatrix} -\gamma_h f' \\ -\gamma_{nh} f \end{bmatrix} dh.$$

By Cramer's Rule, using that the determinant of the Hessian associated with the optimization problem is positive by the second-order condition, we can then write

$$\frac{d\theta}{dh} \stackrel{s}{=} \begin{vmatrix} -\gamma_h f' & \gamma_n f' \\ -\gamma_{nh} f & \gamma_{nn} f - c'' \end{vmatrix}$$

$$\frac{dn}{dh} \stackrel{s}{=} \begin{vmatrix} \gamma f'' & -\gamma_h f' \\ \gamma_n f' & -\gamma_{nh} f \end{vmatrix},$$

where " $\stackrel{s}{=}$ " denotes equality of sign. Evaluating at $n = \check{n}$ and $h = 0$ gives

$$\frac{d\theta}{dh} \Big|_{n=\check{n}, h=0} \stackrel{s}{=} \begin{vmatrix} 0 & 0 \\ -\gamma_{nh} f & \gamma_{nn} f - c'' \end{vmatrix} = 0,$$

$$\frac{dn}{dh} \Big|_{n=\check{n}, h=0} \stackrel{s}{=} \begin{vmatrix} f'' & 0 \\ 0 & -\gamma_{nh} f \end{vmatrix} = -\gamma_{nh} f f'' < 0.$$

That is, the industry club will apply the same standard, but reduce its club size below \check{n} . \square

Comparative statics for the industry club solution with firm heterogeneity: Parallel to the parameterization of consumer heterogeneity above, add variable h to the $\alpha(n)$ function defined in the text, and consider an increase in firm heterogeneity that “pivots” the $\alpha(n, h)$ function anti-clockwise around $\alpha(\tilde{n}, 0)$. Then

$$\alpha_n(n, h) \begin{cases} = 0 & \text{for all } n, h = 0 \\ > 0 & \text{for all } n, h > 0 \end{cases}$$

$$\alpha_h(n, h) \begin{cases} < 0 & \text{for } n < \tilde{n}, h \geq 0 \\ = 0 & \text{for } n = \tilde{n}, h \geq 0 \\ > 0 & \text{for } n > \tilde{n}, h \geq 0 \end{cases}$$

and

$$\alpha_{nh}(n, h) > 0 \quad \text{for all } n, h \geq 0.$$

Note also that, since

$$A(\tilde{n}, h) = \int_0^{\tilde{n}} \alpha(m, h) dm,$$

we have

$$A(\tilde{n}, 0) = \tilde{n}\alpha(\tilde{n}, 0)$$

and

$$A_h(\tilde{n}, h) = \int_0^{\tilde{n}} \alpha_h(m, h) dm < 0.$$

The industry club’s optimization problem becomes

$$\max_{\theta, n} I = f(\theta) - \frac{A(n, h)}{n}\theta - c(n),$$

with first-order conditions

$$\frac{\partial I}{\partial \theta} = f'(\theta) - \frac{A(n, h)}{n} = 0, \tag{A14}$$

$$\frac{\partial I}{\partial n} = \left[\frac{A(n, h) - \alpha(n, h)n}{n^2} \right] \theta - c'(n) = 0. \tag{A15}$$

Totally differentiating w.r.t. θ , n , and h gives

$$\begin{bmatrix} f'' & (A - \alpha n)/n^2 \\ (A - \alpha n)/n^2 & [-\alpha_n/n - 2(A - \alpha n)/n^3]\theta - c'' \end{bmatrix} \begin{bmatrix} d\theta \\ dn \end{bmatrix} = \begin{bmatrix} A_h/n \\ -[A_h/n^2 - \alpha_h/n]\theta \end{bmatrix} dh.$$

By Cramer's Rule, using that the determinant of the Hessian associated with the optimization problem is positive by the second-order condition, we can then write

$$\frac{d\theta}{dh} \stackrel{s}{=} \begin{vmatrix} A_h/n & (A - \alpha n)/n^2 \\ -[A_h/n^2 - \alpha_h/n]\theta & [-\alpha_n/n - 2(A - \alpha n)/n^3]\theta - c'' \end{vmatrix}$$

$$\frac{dn}{dh} \stackrel{s}{=} \begin{vmatrix} f'' & A_h/n \\ (A - \alpha n)/n^2 & -[A_h/n^2 - \alpha_h/n]\theta \end{vmatrix}$$

If we now evaluate these expressions at $n = \tilde{n}$ and $h = 0$, we obtain

$$\frac{d\theta}{dh} \Big|_{n=\tilde{n}, h=0} \stackrel{s}{=} \begin{vmatrix} A_h/n & 0 \\ -A_h\theta/n^2 & -c'' \end{vmatrix} = -\frac{A_h}{n} c'' > 0,$$

$$\frac{dn}{dh} \Big|_{n=\tilde{n}, h=0} \stackrel{s}{=} \begin{vmatrix} f'' & A_h/n \\ 0 & -A_h\theta/n^2 \end{vmatrix} = -\frac{A_h\theta}{n^2} f'' < 0.$$

It follows that when producers are heterogeneous, the industry club will increase the standard and reduce the club size.