On the Use of Autoregression Models to Estimate Climate Sensitivity

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Abstract. First-order autoregression models (ARMs) have often been used to estimate the value of climate sensitivity (ΔT_{2x}) from the observed near-surface temperature record. Here we compare the estimate of ΔT_{2x} by ARM with the estimate obtained by our simple climate/ocean model (SCM: hemispheric energy-balance-climate/upwelling-diffusion-ocean model). We do this for four radiative forcing models (RFMs): (1) greenhouse gases alone (G), (2) G plus anthropogenic sulfate aerosol (GA), (3) GA plus tropospheric ozone and putative solar irradiance variations (GTAS), and (4) GTAS plus volcano radiative forcing (GTASV). We find that the estimates of ΔT_{2x} by the ARM are systematically biased low compared to the estimates of ΔT_{2x} by SCM, ranging from 24% for GTASV to 63% for G. Analysis of the ARM results show that they are equivalent to those given by the simplest climate/ocean model: an energy-balance-climate/mixed-layer-ocean model, with mixed-layer depth h, vertically uniform temperature and no heat exchange with the deeper ocean. The ARM attempts to reproduce all the changes in the observed temperature record, both the rapidly varying changes and the slowly varying changes. To do so the ARM considerably reduces the heat capacity of the climate system such that h ranges from 38.7 m for GA to 185.7 m for GTASV. The corresponding e-folding time for the results of the ARM range from 2.2 years for GTAS to 5.1 years for GTASV. These characteristic times would result in equilibration to a CO_2 doubling in 8.8 years and 20.4 years. These equilibration times are two orders of magnitude smaller than those obtained by the SCM and by coupled atmosphere-ocean general circulation models. Filtering the observed temperatures with running-mean filters to remove their rapidly varying changes improves the performance of the ARM. However, the optimum filter length (OFL) depends on RFM, with values ranging from 7 years for G to 30 years for GA. Because the OFL in general increases with ΔT_{2x}, which is a priori unknown, it is not possible to choose the OFL, except possibly in an iterative manner. Accordingly, it is recommended that the ARM not be used to estimate ΔT_{2x}.

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1. Introduction

The value of the climate sensitivity – the change in global-average near-surface temperature (NST), ΔT_{2x}, resulting from a radiative forcing due to a doubling of the preindustrial carbon dioxide concentration, F_{2x} – has frequently been estimated using the first-order autoregression model (ARM) (e.g., Miles and Gildersleeves, 1977; Tol and Vellinga, 1998),

$$\delta T(t) = b_o + b_1 \delta T(t-1) + b_2 F(t) + \varepsilon(t) \quad , \tag{1}$$

where $F(t)$ is the radiative forcing – the change in the net downward radiation at the tropopause due to increases in greenhouse gases, aerosols and other factors such as solar-irradiance changes and volcanoes, allowing for equilibration of the temperature of the overlying atmosphere; and $\varepsilon(t)$ is the difference between the observed and fitted temperatures. The regression coefficients, b_i, are estimated using $\delta T(t) = \delta T_{obs}(t)$, the observed departure of global-mean NST in year t from a 30-year mean. This ARM is characterized by an e-folding time,

$$\tau = \frac{1}{1 - b_1} \quad , \tag{2}$$

and climate sensitivity,

$$\left(\Delta T_{2x}\right)_{ARM} = b_2 \tau\, F_{2x} \quad . \tag{3}$$

Here we show that $\left(\Delta T_{2x}\right)_{ARM}$ is biased low compared to $\left(\Delta T_{2x}\right)_{SCM}$, the estimate obtained using a simple, physically based climate/ocean model (SCM).

2. Estimating ΔT_{2x} Using a Simple Climate/Ocean Model

The original, global version of the SCM was developed by Schlesinger, based on the model's original formulation by Hoffert *et al.* (1980), and was used by Schlesinger and colleagues to simulate the global-mean temperature evolution for the different greenhouse-gas scenarios of the

IPCC 1990 report (Bretherton *et al*., 1990), and for greenhouse-policy studies (Schlesinger and Jiang, 1991; Hammitt *et al*., 1992; Schlesinger, 1993; Lempert and Schlesinger, 2000, 2002; Lempert *et al*., 1994, 1996, 2000). A hemispheric version of the model was developed to study the influence on the climate system of anthropogenic sulfate aerosol (Schlesinger *et al*., 1992) and putative solar-irradiance variations (Schlesinger and Ramankutty, 1992), and has been used to discover a 65-70 year oscillation in observed surface temperatures for the North Atlantic Ocean and its bordering continental regions (Schlesinger and Ramankutty, 1994a,b; Schlesinger and Ramankutty, 1995). A hemispheric version of the model that explicitly calculates the individual temperature changes over land and ocean in each hemisphere (Ramankutty, 1994) has been used to investigate the influence on climate of volcanoes (Ramankutty, 1994) and the sun (Schlesinger and Ramankutty, 1992), to estimate the causes of climate change from 1856 to 1997 (Andronova and Schlesinger, 2000), and to objectively estimate the probability distribution for ΔT_{2x} (Andronova and Schlesinger, 2001).

The model determines the changes in the temperatures of the atmosphere and ocean, the latter as a function of depth from the surface to the ocean floor (Schlesinger *et al*., 1997). In the model (Figure 1), the ocean is subdivided vertically into 40 layers, with the uppermost being the mixed layer. Also, the ocean is subdivided horizontally into a polar region where bottom water is formed, and a nonpolar region where there is upwelling. In the nonpolar region, heat is transported upwards toward the surface by the water upwelling there and downwards by physical processes whose effects are treated as an equivalent diffusion. Heat is also removed from the mixed layer in the nonpolar region by a transport to the polar region and downwelling toward the bottom, this heat being ultimately transported upward from the ocean floor in the nonpolar region. The atmosphere in each hemisphere is subdivided into the atmosphere over the ocean and the atmosphere over land, with heat exchange between them.

For each radiative-forcing model (RFM) the changes in global-mean ((NH+SH)/2) near-surface temperature, $\Delta T_{GL}^{sim}(t)$, and in the interhemispheric (NH–SH) near-surface temperature difference, $\Delta T_{HD}^{sim}(t)$, were calculated from 1765 through 1997 by the simple climate model for many prescribed values of ΔT_{2x} and the direct (clear-sky) ASA radiative forcing in reference year 1990, $\Delta F_{ASA}^{dir}(1990)$. The simulated departures in global-mean near-surface temperature, $\delta T_{GL}^{sim}(t) = \Delta T_{GL}^{sim}(t) + C_{GL}$, and in the interhemispheric near-surface temperature difference, $\delta T_{HD}^{sim}(t) = \Delta T_{HD}^{sim}(t) + C_{HD}$, were compared with the corresponding observed temperature departures from the 1961-1990 means (Jones *et al*., 1999), $\delta T_{GL}^{obs}(t)$ and $\delta T_{HD}^{obs}(t)$, from 1856 through 1997, with the constants, C_{GL} and C_{HD}, determined to minimize the individual root-mean-square (RMS) differences between $\delta T_{GL}^{sim}(t)$ and $\delta T_{GL}^{obs}(t)$, and $\delta T_{HD}^{sim}(t)$ and $\delta T_{HD}^{obs}(t)$ (Schlesinger and Ramankutty; 1992; Schlesinger *et al*., 1992; Andronova and Schlesinger, 2000, 2001). Maximum-likelihood values of ΔT_{2x} and $\Delta F_{ASA}^{dir}(1990)$ – hence the total (all-sky) ASA radiative forcing $\Delta F_{ASA}(1990) = 3.67\ \Delta F_{ASA}^{dir}(1990)$ (Harvey *et al*., 1997) – were determined by simultaneously minimizing the RMS difference between $\delta T_{GL}^{sim}(t)$ and $\delta T_{GL}^{obs}(t)$ and between $\delta T_{HD}^{sim}(t)$ and $\delta T_{HD}^{obs}(t)$. This was done separately for monthly and annual observed temperatures, with negligible differences in the results.

3. Comparison of ΔT_{2x} Estimated by the ARM and SCM

Table 1 presents the estimates of ARM and SCM for four radiation forcing models (RFMs): (1) greenhouse gases alone (G), (2) G plus anthropogenic sulfate aerosol (GA), (3) GA plus tropospheric ozone and the solar irradiance variations of Lean *et al.* (1995) (GTAS), and (4) GTAS plus the volcano radiative forcing of Andronova *et al.* (1999) (GTASV). (For additional information about these RFMs, see Andronova and Schlesinger, 2001.) It is seen that $(\Delta T_{2x})_{ARM}/(\Delta T_{2x})_{SCM}$ ranges from 0.24 for GTASV to 0.63 for G and generally decreases with increasing $(\Delta T_{2x})_{SCM}$. Thus $(\Delta T_{2x})_{ARM}$ is systematically biased low compared to $(\Delta T_{2x})_{SCM}$, with the size of the bias depending on RFM.

4. Analysis of the ARM Results

The reason for this can be seen in Fig. 2 wherein are presented $\delta T_{SCM}(t)$ and $\delta T_{ARM}(t)$ for GTAS, the latter defined by Eq. (1) with $\varepsilon(t)$ taken as zero for all t, together with $\delta T_{obs}(t)$. It can be seen that the ARM attempts to reproduce all the changes in $\delta T_{obs}(t)$, both the rapidly varying changes and the slowly varying changes, while the SCM reproduces only the slowly varying changes. To reproduce the rapidly varying changes, the ARM obtains e-folding times τ that are extremely short (Table 1), ranging from 2.2 years for GTAS to 5.1 years for GTASV, which in turn result in small values of $(\Delta T_{2x})_{ARM}$ (see Eq. (3)). The ARM does this by obtaining values of b_1 that are not very close to unity (see Eq. (2)). These short e-folding times would lead the climate system forced by an instantaneous CO_2 doubling to reach within 2% of ΔT_{2x} within $(4\tau =)$ 8.8 and 20.4 years, respectively. These are very unrealistically short equilibration times compared with the equilibration times of coupled atmosphere-ocean general circulation models (Bryan, 1984) and the SCM, which require more than a thousand years to equilibrate (Fig. 3).

To achieve these short e-folding times, the ARM effectively reduces the heat capacity of the ocean such that the climate system can react very rapidly. To see this, we can interpret the results of the ARM in terms of the simplest physical climate/ocean model – that for the upper well-mixed layer of the ocean, with depth h, vertically uniform temperature and no heat exchange with the deeper ocean. Such a climate/mixed-layer-ocean model is governed by

$$\rho ch \frac{d\Delta T}{dt} = -\frac{F_{2x}}{\Delta T_{2x}} \Delta T(t) + F(t) \quad , \tag{4}$$

where $\Delta T(t) = \delta T(t) - b_o$ is the temperature change from some initial time t_o when $\delta T(t_o) = b_o$, and ρ and c are the density and specific heat of sea water. After finite differencing (Euler forward), Eq. (4) can be written as Eq. (1), with

$$b_2 = \frac{\Delta t}{\rho c h} \quad , \tag{5}$$

where Δt is the time step (1 year) and

$$b_1 = 1 - b_2 \left[\frac{F_{2x}}{\left(\Delta T_{2x}\right)_{ARM}} \right] \quad , \tag{6}$$

the latter as given by Eqs. (2) and (3). We can use Eq. (5) to calculate the mixed-layer-ocean depths shown in Table 1 for the ARM estimates. It is seen that h ranges from 38.7 m for GA to 185.7 m for GTASV. Thus the ARM representation of the observed NST departures is equivalent to that for a climate system whose heat capacity includes only that of the oceanic mixed layer.

5. Improving the ARM Results

One way to try to overcome this unrealistic behavior of the ARM would be to remove the rapidly varying changes in the observed NST record. By the analysis above, doing so should increase h by decreasing b_2 (Eq. 5) and increase τ by increasing b_1 towards unity (Eq. 1). Figure 4 shows h, τ and $\left(\Delta T_{2x}\right)_{ARM} / \left(\Delta T_{2x}\right)_{SCM}$ for GTAS as a function of the length of a simple running-mean filter. It is seen that h, τ and $\left(\Delta T_{2x}\right)_{ARM} / \left(\Delta T_{2x}\right)_{SCM}$ increase with the length of the filter. The filter length for which $\left(\Delta T_{2x}\right)_{ARM} / \left(\Delta T_{2x}\right)_{SCM} = 1$, the optimum filter length (OFL), is 25 years for GTAS. The OFLs for the four RFMs are shown in Table 1. It is seen that the OFL depends on RFM, with values ranging from 7 years for G to 30 years for GA. It can also be seen that except for GTASV, the OFL increases with ΔT_{2x}.

6. Conclusion

We have found that the ARM systematically underestimates the value of ΔT_{2x} in comparison with the value of ΔT_{2x} estimated by a simple climate/ocean model (SCM), each model fitting the observed record of near-surface temperature (NST) changes from 1856-1997. The ARM does this because it attempts to fit not only the slowly varying changes in NST, but also the rapidly varying changes. To do so, the ARM reduces the heat capacity of the climate system, represented by the depth of ARM's mixed-layer ocean. This results in a characteristic response time for ARM that is two orders of magnitude smaller than the characteristic response time of coupled atmosphere/ocean general circulation models and the SCM. We have attempted to improve the performance of the ARM by filtering the NST. However, we found that the optimum filter length (OFL) required for $\left(\Delta T_{2x}\right)_{ARM} / \left(\Delta T_{2x}\right)_{SCM} = 1$ depends on the radiative forcing model and, in general, increases with ΔT_{2x}. Since ΔT_{2x} is a priori unknown, it is not possible to choose the OFL, except possibly in an iterative manner. Accordingly, we recommend that the ARM of Eq. (1) not be used to estimate ΔT_{2x}.

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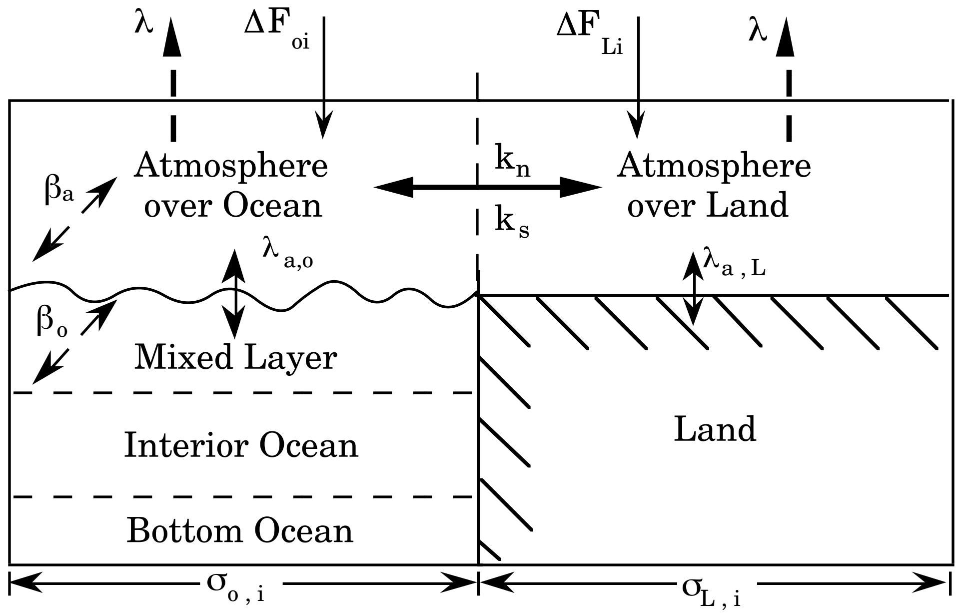
Acknowledgement. This material is based upon work supported by the National Science Foundation under Award No. ATM-0084270. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation.

Table 1. Estimates of ARM and SCM for four radiative forcing models.

	Radiative Forcing Model			
Quantity	G	GA	GTAS	GTASV
b_o (°C)	– 0.167	– 0.166	– 0.179	– 0.0386
b_1	0.583	0.590	0.539	0.803
b_2 (°C/Wm^{-2})	0.120	0.198	0.177	0.0412
τ (Years)	2.40	2.44	2.17	5.08
h (m)	63.9	38.7	43.2	185.7
$\left(\Delta T_{2x}\right)_{ARM}$ (°C)	1.07	1.79	1.42	0.78
$\left(\Delta T_{2x}\right)_{SCM}$ (°C)	1.69	5.74	2.97	3.20
$\left(\Delta T_{2x}\right)_{ARM} / \left(\Delta T_{2x}\right)_{SCM}$	0.63	0.31	0.48	0.24
Optimum Filter Length (Years)	7	30	25	7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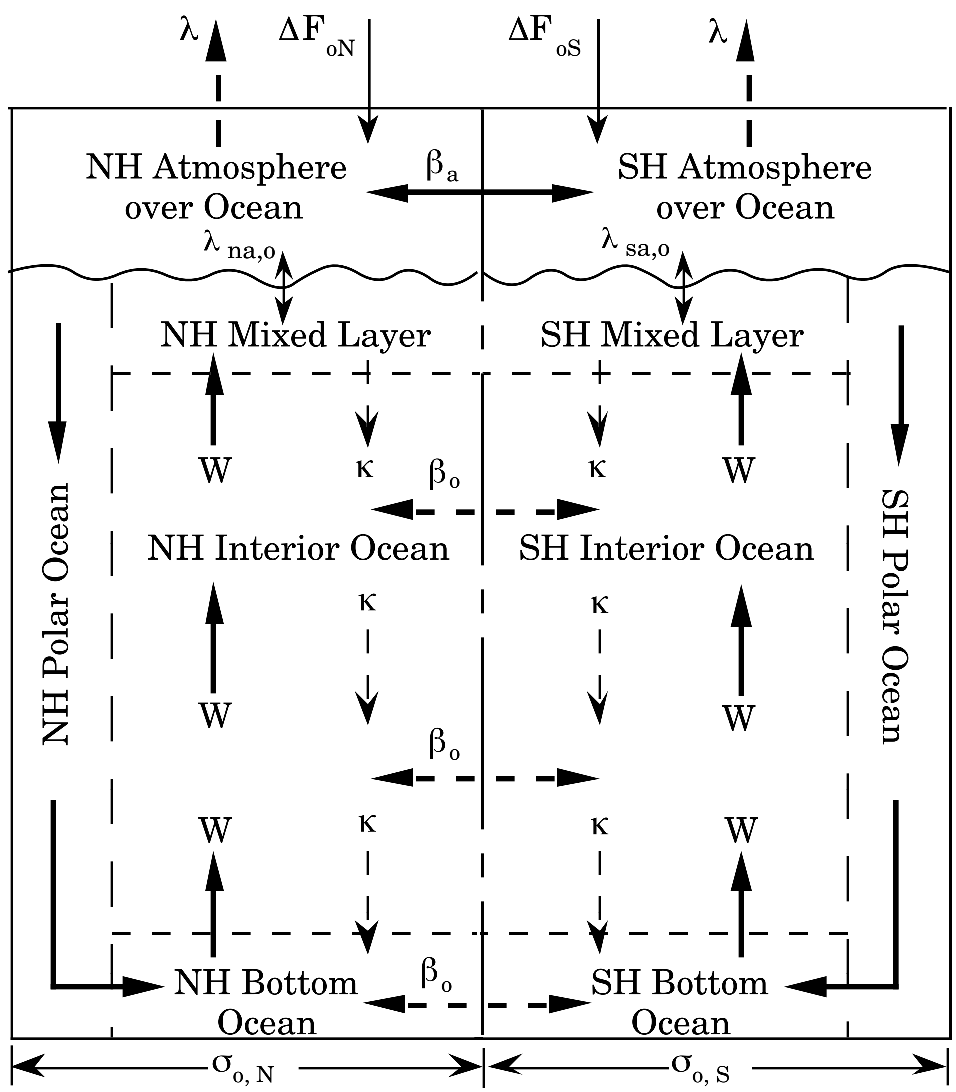
Mixed Layer
Interior Ocean
Land
Bottom Ocean
σ o , i
σ L , i


λ
ΔF oN
ΔF oS
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NH Atmosphere over Ocean
β a
SH Atmosphere over Ocean
λ na,o
λ sa,o
NH Mixed Layer
SH Mixed Layer
W
κ
β o
κ
W
NH Polar Ocean
NH Interior Ocean
SH Interior Ocean
SH Polar Ocean
κ
κ
W
W
β o
W
κ
κ
W
NH Bottom Ocean
β o
SH Bottom Ocean
σ o, N
σ o, S

Figure 1. Schematic diagram of the EBC/UDO model. The top panel shows the general structure of the model, and the bottom panel shows a vertical cross-section through the oceanic part. The symbols by the arrows indicate the following physical processes: ΔF_{Li} and ΔF_{oi}, tropopause radiative forcing in hemisphere i over land and ocean, respectively; λ, radiative-plus-feedback temperature response of the climate system; k, atmospheric land-ocean heat exchange; β_a, atmospheric interhemispheric heat exchange; $\lambda_{a,o}$, air-sea heat exchange; $\lambda_{a,L}$, air-land heat exchange; β_o, oceanic interhemispheric heat exchange; W, vertical heat transport by upwelling; κ, vertical heat transport by diffusion. The quantities $\sigma_{L,i}$ and $\sigma_{L,o}$ denote the fractions of hemisphere i covered by land and ocean, respectively.

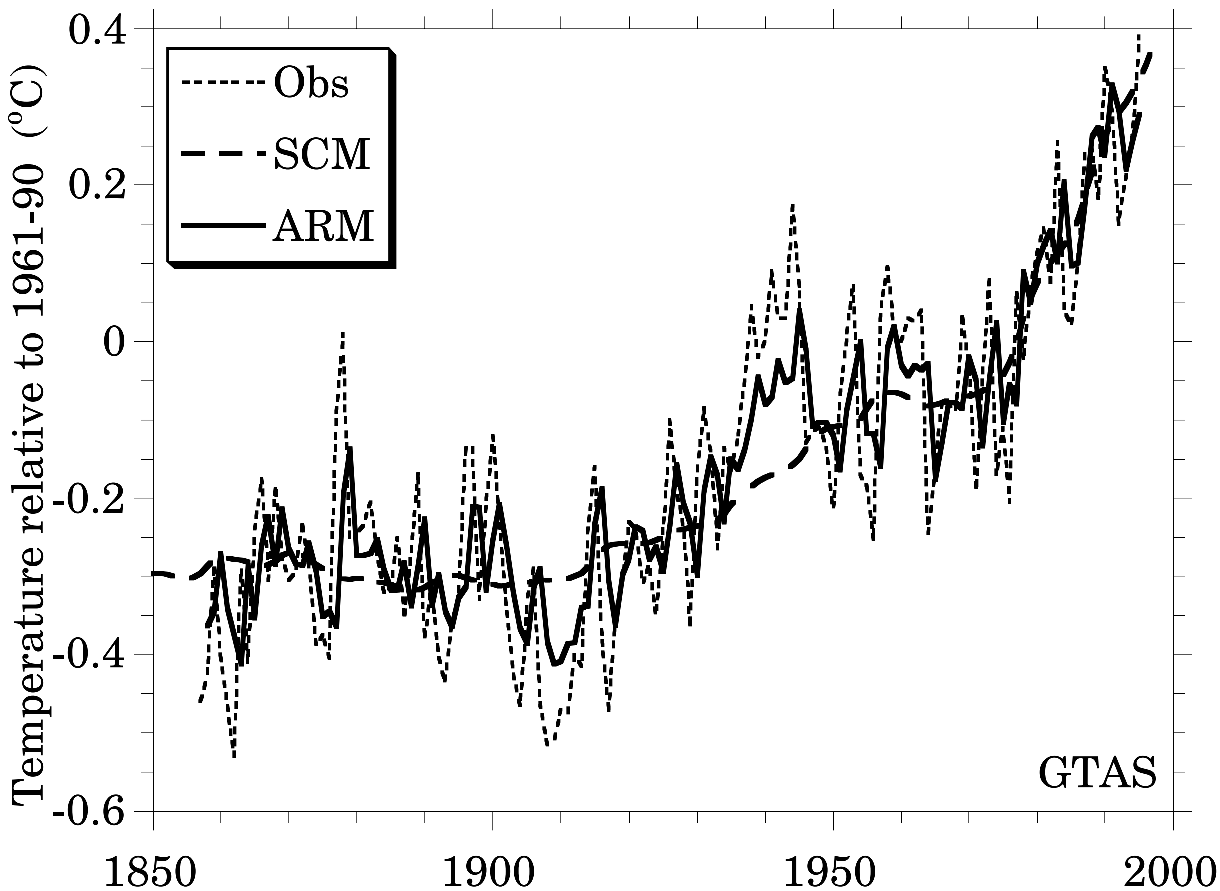

Figure 2. The temperature departures simulated by the ARM (solid line) and SCM (long dashed line) for GTAS, together with observed temperature departure (dashed line), each relative to 1961-90.

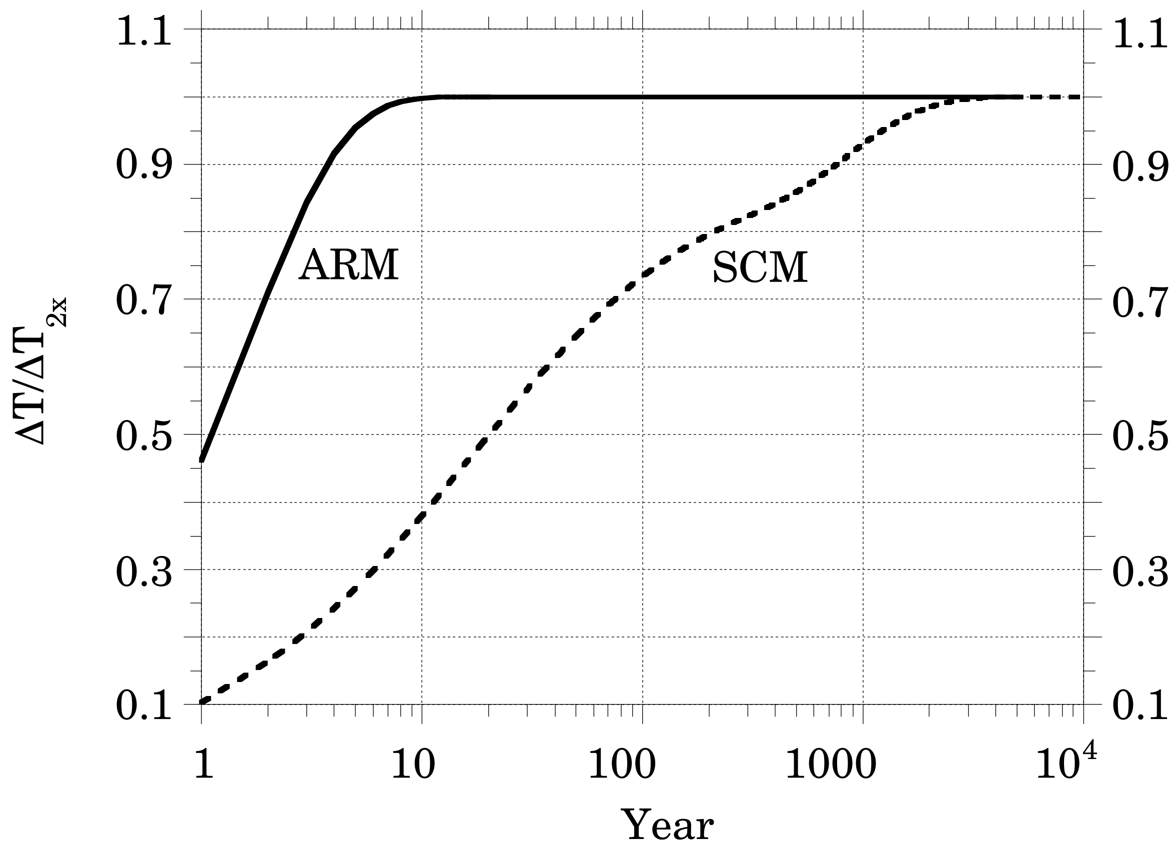

Figure 3. Temperature responses, normalized by ΔT_{2x}, for the ARM and SCM to an instantaneous doubling of the CO_2 concentration, using the parameter values shown in Table 1 for GTAS.

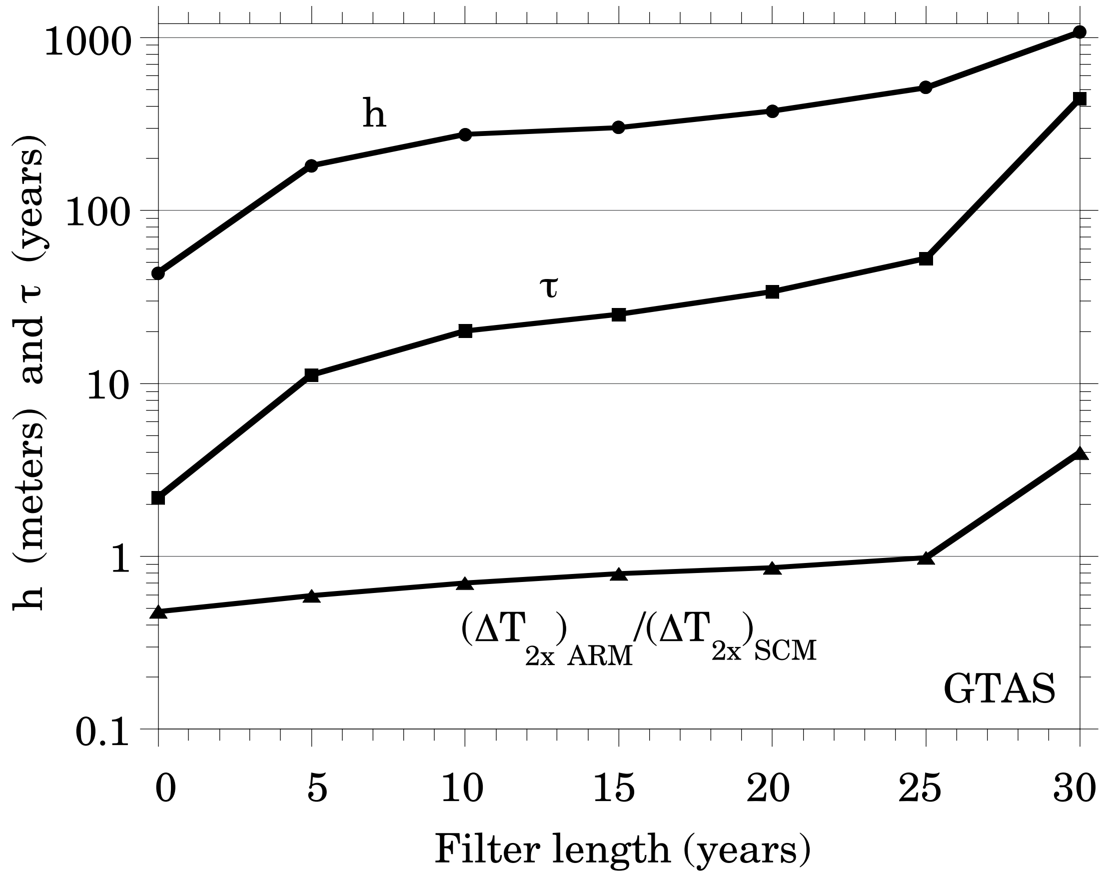

Figure 4. Values of h, τ and $\left(\Delta T_{2x}\right)_{ARM}/\left(\Delta T_{2x}\right)_{SCM}$ for GTAS as a function of the length of a running-mean filter.