Automated spectro-goniometer: A spherical robot for the field measurement of the directional reflectance of snow

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We describe an automated spectro-goniometer (ASG) that rapidly measures the spectral hemispherical-directional reflectance factor (HDRF) of snow in the field across the wavelength range 0.4<\lambda<2.5 \text{\mu m}. Few measurements of snow’s HDRF exist in the literature, in part caused by a lack of a portable instrument capable of rapid, repeatable sampling. The ASG is a two-link spherical robot coupled to a field spectroradiometer. The ASG is the first revolute joint and first automated field goniometer for use over snow and other smooth surfaces. It is light enough (~50 kg) to be portable in a sled by an individual. The ASG samples the HDRF at arbitrary angular resolution and 0.5 Hz sampling rate. The arm attaches to the fixed-point frame 0.65 m above the surface. With vertical and oblique axes, the ASG places the sensor of the field spectroradiometer at any point on the hemisphere above a snow target. In standard usage, the ASG has the sun as the illumination source to facilitate in situ measurements over fragile surfaces not easily transported to the laboratory and to facilitate simultaneous illumination conditions for validation and calibration of satellite retrievals. The kinematics of the ASG is derived using Rodrigues’ formula applied to the 2 degree-of-freedom arm. We describe the inverse kinematics for the ASG and solve the inverse problem from a given view angle to the necessary rotation about each axis. Its two-dimensional hemispheric sampling space facilitates the measurement of spectral reflectance from snow and other relatively smooth surfaces into any direction. The measurements will be used to validate radiative transfer model results of directional reflectance and to validate/calibrate directional satellite measurements of reflectance from these smooth surfaces. © 2003 American Institute of Physics. [DOI: 10.1063/1.1626011]

I. INTRODUCTION

Distributed snowmelt models use snow-covered area, grain size, albedo, and snow water equivalent for input and validation. Estimates of all but snow water equivalent are available from optical (solar) remote sensing. Algorithms for retrieving these snow properties have been based on the simplifying assumption that snow reflects solar radiation isotropically. We know that snow has an anisotropic reflectance distribution that depends on wavelength, grain size, and grain morphology, but our understanding is hampered by the lack of measurements that cover the full spectral and angular distribution.

Satellites sample only a part of this spectral hemispherical-directional reflectance factor (HDRF) that has been convolved with interactions on heterogeneous terrain.

The assumption of isotropic reflectance will frequently produce errors in quantitative retrievals of snow physical properties.

Recent increases in computational power and data storage have enabled scientists to develop and refine quantitative retrievals of snow properties by incorporating the anisotropic reflectance distribution of snow. Existing models for the HDRF of snow have not been compared with a comprehensive sampling of the HDRF and its dependence on solar geometry, wavelength, grain size distribution, grain morphology, liquid water content, and impurities. To meet this need, we have developed an automated spherical robot for use with a high spectral resolution field spectroradiometer (Fig. 1).

We describe the automated spectro-goniometer (ASG) in terms of its constraints, spectral range and resolution, kinematics, sampling protocols, and specifications. The ASG is unique in that it is the first revolute joint goniometer and the first automated field goniometer for snow measurements. It is light enough (~50 kg) to be portable by an individual via sled. In this section, we define the hemispherical-directional

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reflectance factor (HDFR) and the hemispherical-directional reflectance distribution function (HDRDF), review previous measurements of snow HDFR, and review other available goniometers for field measurements of angular reflectance. In Sec. II, we present system constraints and the spectrometer specifications. Section III describes the kinematics of the automated spectro-goniometer and their closed-form inversion. In Sec. IV, we describe the motion control configuration. In Sec. V, we describe the angular calibration of the ASG. Section VI presents the calculation of HDRF from the ASG. Preliminary results and discussion are given in Sec. VII.

A. Hemispherical-directional reflectance factor

The commonly used, dimensionless form of the angular distribution of reflectance is the hemispherical-directional reflectance factor (HDFR)\(^7,8\)

\[
\text{HDFR}_{\lambda}(\theta_0, \phi_0; \theta_r, \phi_r) = \frac{L_{\lambda}(\theta_r, \phi_r)}{\mu_0 E_{\lambda,\text{dir}}(\theta_0, \phi_0) + E_{\lambda,\text{diff}}(2\pi)} \frac{\pi L_{\lambda}(\theta_r, \phi_r)}{\mu_0 E_{\lambda,\text{dir}}(\theta_0, \phi_0) + E_{\lambda,\text{diff}}(2\pi)},
\]

where \(L_{\lambda}\) is spectral radiance (W m\(^{-2}\)sr\(^{-1}\)\(\mu\)m\(^{-1}\)), \(E_{\lambda,\text{dir}}\) is spectral irradiance (W m\(^{-2}\)\(\mu\)m\(^{-1}\)) onto a plane normal to the beam of the Sun, \(E_{\lambda,\text{diff}}\) is diffuse spectral irradiance (W m\(^{-2}\)\(\mu\)m\(^{-1}\)) from the overlying hemisphere (solid angle of 2\(\pi\) steradians) less the solid angle containing the solar disk, \(\theta_0\) is the zenith angle of direct beam irradiance, \(\phi_0\) is the azimuth angle of direct beam irradiance, \(\theta_r\) is the zenith angle of reflected radiance, \(\phi_r\) is the azimuth angle of reflected radiance, and \(\mu_0\) is the cosine of \(\theta_0\). The HDFR is the ratio of radiance reflected into the direction \(\theta_r, \phi_r\) to the radiance that would be reflected into any direction by a perfectly reflecting Lambertian (i.e., isotropically reflecting) surface for the given direct and diffuse irradiance.

The hemispherical-directional reflectance distribution function (HDRDF) of a surface describes the magnitude of reflected radiance into a direction relative to the irradiance from a given direction plus the hemispherical diffuse irradiance (Fig. 2)

![Fig. 2. Representation of hemispherical-directional reflectance.](image)

\[
\text{HDRDF}_{\lambda}(\theta_0, \phi_0; \theta_r, \phi_r) = \frac{\text{HDFR}(\theta_0, \phi_0; \theta_r, \phi_r)}{\pi}. \quad (2)
\]

The HDRDF has units of inverse steradians (sr\(^{-1}\)) and is analogous to the bidirectional reflectance distribution function.\(^7\)

B. Previous snow HDRF measurements

The literature describing the HDRF of snow is sparse. Previous efforts to characterize the HDRF of snow have been limited with respect to one or more of angular resolution and range, spectral resolution and spectral range, speed of acquisition, and quantitative documentation of snow properties.\(^9\)–\(^11\) Steffen\(^10\) measured the HDRF of snow for the wavelength range 0.5 \(\mu\)m\(<\lambda\)<1.0 \(\mu\)m, up to the shorter wavelength end of the spectral region in which the reflectance of snow is most sensitive to grain size. Leroux et al.\(^2\) compared a model with measurements of snow HDRF accompanied by detailed quantification of grain size distribution and grain morphology, but they reported measurements only for \(\lambda = 1.68 \mu\)m.

C. Goniometers

Goniometers are instruments that measure angles. The word “goniometer” is derived from the Greek word “gonia” meaning angle. Remote sensing-specific field goniometers have been developed primarily for the study of the HDRF of vegetative covers. Other fields such as optical properties of materials,\(^13\) computer graphics modeling,\(^14\) and colorimetry\(^15\) use goniophotometers in a variety of configurations. Because the ASG is specifically a field goniometer, we review the field goniometers used in remote sensing applications and refer the reader to more comprehensive works describing goniophotometry in other fields.\(^16\)

The first goniometric field instrument regularly used for remote sensing and radiative transfer analysis of vegetation
was the portable apparatus for rapid acquisition of bidirectional observations of the land and atmosphere (PARABOLA). The PARABOLA, a dual-axis up-and-down-looking three-band radiometer, rides a tram cableway over a target such as a vegetation canopy. In geometric terms, PARABOLA looks from the center of a hemisphere out to discrete points on the hemisphere. This geometry dictates that PARABOLA views different targets at each pair of zenith and azimuth view angles and so configured, relies on the assumption that the surface cover is spatially homogeneous. The latest version of PARABOLA is the PARABOLA III, described by Bruegge et al.

The Remote Sensing Laboratories at the University of Zurich developed the field goniometer system (FIGOS) for hyperspectral measurement of the hemispherical-directional reflectance distribution of vegetation and soils. FIGOS has a 2 m radius zenith arc with a tram that carries a GER Corporation GER-3700 spectroradiometer, and an azimuth support track. FIGOS can cover the hemisphere at 15° angular sampling in approximately 18 min, making 66 measurements. While the FIGOS weighs 230 kg and requires two technicians for assembly and operation, it has advantages over PARABOLA in that it measures the complete optical spectrum at high spectral resolution and views the same target for all spectra.

Aoki et al. sampled snow HDRF and albedo with a gonostage coupled to a tripod. This instrument is similar to PARABOLA in that it views the surface from the center of the hemisphere and assumes spatial homogeneity. While this instrument is inexpensive to manufacture and deploy, the occultation of many view zeniths and azimuths by the tripod prevents complete HDRF sampling.

Of these goniometric instruments, the PARABOLA and FIGOS are automated. The ASG described here simplifies the mechanism design by employing revolute joints with a relatively inconsequential increase in computational complexity of the kinematics.

### II. SYSTEM DESCRIPTION

A comprehensive characterization of snow HDRF must address two sets of parameters, one set corresponding to instrumentation and the other corresponding to snow and atmospheric characterization. The instrument parameters are spectral range and resolution, acquisition speed, angular range and resolution, angular accuracy, and acquisition speed (if using the sun as an illumination source). Characterization of the snow and atmosphere includes the spectral ratio of diffuse to direct irradiance, grain size distribution, grain morphology and its distribution, snow liquid water content, density, and impurities. This section discusses the instrumentation.

#### A. Field spectrometer

The automated spectro-goniometer manipulates the sensor of an Analytical Spectral Devices FieldSpec FR spectroradiometer (ASD-FR). The ASD-FR has a 512 element Si photodiode array to cover the wavelength range 0.35–1.0 μm, and two separate TE-cooled, graded index InGaAs photodiodes to cover the wavelength range 1.0–2.5 μm. We show the specifications for the ASD-FR in Table I and the reader may find further specifications at http://www.asdi.com. Of note here are the complete spectral range (0.35–2.5 μm), high spectral resolution (0.003–0.010 μm), and the rapid spectrum acquisition (10 spectra/s and an average spectrum automatically recorded at 1 Hz). With the rapid spectrum acquisition, the ASG may access a dense grid of points on the hemisphere above the snow target with little change in solar geometry.

#### B. Configuration

The ASG has two independently controlled motorized arms serially attached to a boom that extends from a circular base (Fig. 3). The boom has an attachment point for the first motor and the upper joint housing that encloses a radially pre-loaded anti-backlash worm wheel, worm shaft, and ball bearing assembly (Fig. 4). Two ball bearings support the worm wheel. The second motor attaches to the end of the

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**TABLE I. Specifications for Analytical Spectral Devices FR FieldSpec Spectroradiometer.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectral range</td>
<td>0.35–2.5 μm</td>
</tr>
<tr>
<td>Spectral resolution</td>
<td>0.003 μm (0.35&lt;λ≤1.0 μm)</td>
</tr>
<tr>
<td></td>
<td>0.010 μm (1.0&lt;λ≤2.5 μm)</td>
</tr>
<tr>
<td>Spectrum sampling rate</td>
<td>10 spectra/s</td>
</tr>
<tr>
<td>Spectrum recording rate</td>
<td>1 spectrum/s (max. rate)</td>
</tr>
<tr>
<td>Weight</td>
<td>9.2 kg</td>
</tr>
<tr>
<td>Optic cable length</td>
<td>2.5 m</td>
</tr>
<tr>
<td>Fore optics</td>
<td>1°, 4°, 8°</td>
</tr>
<tr>
<td>Noise-equivalent change in reflectance (NEDL)</td>
<td>3.7×10^-10 W/cm²/nm/sr @ 700 nm²</td>
</tr>
</tbody>
</table>

*Provided on the Analytical Spectral Devices webpage www.asdi.com.*

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**FIG. 3. Geometry for the automated spectro-goniometer.** The elbow axis \( k_5 \), lies in the \( x-z \) plane, with a zenith angle of \( \beta = 46.5° \) off of the \( z \) axis. The three vectors \( z \) (vertical axis), \( k_5 \) (the oblique axis), and \( u \) (the view vector) intersect at the hemisphere origin, \( \theta_1 \) and \( \theta_2 \) are the rotations about \( z \) and \( k_0 \), respectively. The elbow axis \( k_4 \) lies in the \( x-z \) plane when \( \theta_1 = 0 \).
first arm in a lower joint housing (Fig. 5). This housing contains the second worm gear, worm shaft, and a single ball bearing.

The ASD-FR optic cable end point attaches to the end of the lower arm of the ASG (Fig. 6). The optic cable points at the center of the base circle from the scanning space of the hemisphere above this center discussed in Sec. III. Brushless servomotors drive rotation about the vertical and oblique axes, placing the optic cable end point at any point on the hemisphere. Hence, the ASG has arbitrary angular range. In design and manufacture, we specified angular accuracy of 2°. The Pittman motor/encoders described below have 2000 counts per revolution which, combined with 72 and 95 teeth worm gears, provide potential angular resolutions of 0.0025° and 0.0019°, although joint flexibility and friction prevent the attainment of these resolutions.

III. KINEMATICS

In this section, we present the kinematics of the mechanism. The kinematics of the 2 degree-of-freedom robot describe the view vector, $u$, as a function of the joint angles $\theta_1$ and $\theta_2$ (Fig. 3).

The fore-optic mount moves in the hemisphere of constant radius (0.65 m) about the target. The kinematics of the ASG are derived using Rodrigues’ formula\textsuperscript{22} for a spherical displacement of a rigid body. The protocol for use of the ASG is determined by a prescribed set, $U$, of viewing angles or view vectors $u$. Hence, we must solve the inverse kinematics problem: given $u$, find $\theta_1$, $\theta_2$ which are the respective rotations about $z$ and $k_0$. Except at full reach and full retraction of the ASG arms, there are two solutions to the inverse kinematics problem.

All joint rotations are about axes that pass through the center, $O$, of the hemisphere which is coincident with the target. If $k$ is an axis of rotation, the $3 \times 3$ rotation matrix $R_k$ that represents the rotation about $k$ by an angle $\theta$ is given by

$$R_k(\theta) = (1 - \cos \theta)k^T + \cos \theta I + \sin \theta [k \times], \quad (3)$$

where

$$[k \times] =
\begin{bmatrix}
0 & -k_3 & k_2 \\
 k_3 & 0 & -k_1 \\
- k_2 & k_1 & 0 
\end{bmatrix}, \quad (4)$$

$k^T$ is the transpose of $k$, and $\theta$ is the angle of rotation about $k$. Equation (3) is known as Rodrigues’ formula.

Figure 3 depicts the ASG in a general position. However, when both joint angles are zero, the spectrometer fore-optic points in the $-z$ direction and the second joint axis $k_0$ lies in the $x$-$z$ plane. From Fig. 3,

$$k_0 =
\begin{pmatrix}
\sin \beta \\
0 \\
\cos \beta
\end{pmatrix} \cdot \quad (5)$$

When the ASG elbow rotates $\theta_2$, the new orientation of the fore-optic is
\[ R_{k_0}(\theta_2)(-z), \]  
\[ (6) \]

where \( R_{k_0}(\theta_2) \) is the rotation about \( k_0 \) by \( \theta_2 \). When the ASG shoulder rotates \( \theta_1 \) about \(-z\), the orientation becomes

\[ u = R_z(\theta_1)R_{k_0}(\theta_2)(-z), \]  
\[ (7) \]

where the rotation matrices can be computed from Eq. (3).

We must now solve Eq. (7) for \( \theta_1 \) and \( \theta_2 \) given \( u \). Multiply Eq. (7) on the left by \( z^T \) to get

\[ z^Tu = z^TR_z(\theta_1)R_{k_0}(\theta_2)(-z) \]  
\[ (8) \]

and since \( z^TR_z = z^T \) (\( z \) is a left eigenvector of a rotation about \( z \)), we have

\[ z^Tu = z^TR_k(\theta_2)(-z) \]  
\[ (9) \]

which may be solved for \( \theta_2 \).

Substituting Eq. (5) into Eq. (3) gives

\[ R_{k_0}(\theta_2)(-z) = -\left[ (1 - \cos \theta_2)k_0k_0^Tz + \cos \theta_2z \right. \]  
\[ + \sin \theta_2[k_0 \times (z)](-z) \left. \right] \]

\[ = \left[ (1 - \cos \theta_2) \begin{pmatrix} \sin \beta \\ \cos \beta \end{pmatrix} \cos \beta \right. \]  
\[ + \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \sin \theta_2 \begin{pmatrix} 0 \\ -\sin \beta \end{pmatrix} \left. \right] \]

\[ = (\cos \theta_2 - 1) \begin{pmatrix} \sin \beta \cos \beta \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cos \theta_2 \]  
\[ (10) \]

Combining Eqs. (7) and (10) gives

\[ u_z = \cos \theta_2(\cos^2 \beta - 1) \]  
\[ - \cos^2 \beta \]  
\[ (11) \]

and, in turn

\[ \theta_2 = \cos^{-1}\left( \frac{u_z + \cos^2 \beta}{\cos^2 \beta - 1} \right), \]  
\[ (12) \]

where \( u_z = z^Tu \). Since there exist two branches to the \( \cos^{-1} \), there exist generally two solutions to choose from at this point in the calculation. A single branch is used throughout a scan except when occlusion by the arm forces a switch to a different branch.

Given \( \theta_2 \), we now solve for \( \theta_1 \). We define \( w \) as

\[ w = R_{k_0}(\theta_2)(-z) \]  
\[ (13) \]

and solve

\[ u = R_z(\theta_1)w \Rightarrow \left[ \begin{array}{c} u_x \\ u_y \\ u_z \end{array} \right] \]

\[ = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix} \]

\[ \Rightarrow \left[ \begin{array}{c} u_x \\ u_y \end{array} \right] \]

\[ = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} w_x \\ w_y \end{pmatrix}. \]  
\[ (14) \]

We define \( a \) and \( r \) as follows:

\[ \alpha = \arctan_2 \left( \frac{w_y}{w_x} \right), \]

\[ r = \sqrt{w_x^2 + w_y^2}, \]  
\[ (15) \]

so that

\[ w_x = r \cos \alpha, \]

\[ w_y = r \sin \alpha. \]  
\[ (16) \]

Combining these definitions with Eq. (14) yields

\[ \begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{pmatrix} \begin{pmatrix} r \cos \alpha \\ r \sin \alpha \end{pmatrix} \]

\[ \begin{pmatrix} u_x \\ u_y \end{pmatrix} = \begin{pmatrix} r \cos \theta_1 \cos \alpha - r \cos \theta_1 \sin \alpha \\ r \sin \theta_1 \cos \alpha + r \cos \theta_1 \sin \alpha \end{pmatrix} \]

\[ = r \begin{pmatrix} \cos \left( \theta_1 + \alpha \right) \\ \sin \left( \theta_1 + \alpha \right) \end{pmatrix} \Rightarrow \theta_1 + \alpha = \arctan_2 \left( \frac{u_y}{u_x} \right). \]  
\[ (17) \]

Therefore,

\[ \theta_1 = \arctan_2 \left( \frac{u_y}{u_x} \right) - \alpha. \]  
\[ (18) \]

For each solution for \( \theta_1 \), there exists a corresponding solution to Eq. (18). Hence, there are two solutions overall for the pair \( \theta_1, \theta_2 \).

The above parameters for the ASG are given in Table II.

\[ \begin{array}{|c|c|}
\hline
\text{Parameter} & \text{Value} \\
\hline
\beta & 46.5^\circ \\
\hline
k_0 & \left[ \sin 46.5^\circ, 0, \cos 46.5^\circ \right] = [0.725, 0, 0, 0.688] \\
\hline
\text{Hemisphere radius} & 0.65 \text{ m} \\
\hline
\end{array} \]

Given the above parameters, we then calculated the necessary rotations \( \theta_1, \theta_2 \). We chose \( \beta = 46.5^\circ \) so that the arm would be bent when at the largest view zenith angle (80°). This prevents occultation of the sun at the maximum extension of the arm.

At the scale of field instrumentation, occultation of the sun by the instrument must occur when the view zenith and view azimuth angles match those of the sun. Occultation
occurs with the ASG when the fore-optic mount (the end of the robotic arm) aligns with the sun and the target center. For a prescribed set of view zenith and view azimuth angles, we solved for the necessary relative \( \theta_1 \) and \( \theta_2 \) in terms of motor counts (Fig. 7). Rotations about \( \theta_1 \) and \( \theta_2 \) are translated to digital motor counts \( \theta_{1,MC} \) and \( \theta_{2,MC} \), respectively, by the following relationships:

\[
\theta_{1,MC} = \frac{\theta_1 (N_{wg1}) (C_{rev})}{360}, \\
\theta_{2,MC} = \frac{\theta_2 (N_{wg2}) (C_{rev})}{360},
\]

where \( N_{wg1} \) and \( N_{wg2} \) are the respective numbers of teeth on the worm gears and \( C_{rev} \) is the number of motor counts per motor axle revolution. For the ASG, \( N_{wg1} \) and \( N_{wg2} \) are 95 and 72, respectively, and \( C_{rev} \) is 2000.

IV. CONTROL HARDWARE

A summary of the specifications of the ASG is shown in Table III. Pittman Series 3400 brushless servomotors drive rotation about the axes of the ASG. A Galil Motion Control DMC-2400 motor controller and two Advanced Motion Control B12A6 amplifiers control the motors. The integrated system draws 110 V ac power. An autonomous laptop drives the motor controller via a universal serial bus (USB) interface and communicates via a nine-pin RS-232 interface with a laptop that is integrated with the ASD-FR spectroradiometer (Fig. 8). The ASD-FR runs in automatic regular acquisition and sends a pulse to the serial port upon completion of spectrum sampling. This pulse initiates a discrete motion of the ASG arm to the next node on the sampling grid.

The ASD-FR can sample automatically at a 1 s interval. However, running the ASD-FR at a 2 s (0.5 Hz) interval allows the time needed for ASG arm movement (~600 ms for 15° rotation) and vibration settling. The ASG has three fundamental sampling protocols: hemisphere at 10° zenith and azimuth sampling, hemisphere at 15° zenith and azimuth sampling, and 1° zenith sampling in the solar principal plane. The respective sampling times for these protocols are 5:40 (mm:ss), 2:36, and 5:42, assuming symmetry across the solar principal plane.

The greatest limitation of using the sun as an illumination source in the field is change in illumination geometry \((\Delta \theta_0, \Delta \phi_0)\). However, the ASG size and speed facilitate rapid acquisitions that minimize the effects of changes in solar geometry. Figure 9 shows that in the winter and spring months, a 6 min sampling interval results in solar geometry change less than the 2° angular accuracy specification for the ASG. Hence, changes in solar geometry should have negligible effect on the HDRF results. During summer months, the changes in solar geometry are more pronounced at midday. However, the small solar zenith angles at these times result in a more radially symmetric HDRF. Nonetheless, we change to the 15° sampling grid to decrease the sampling time and in turn decrease the change in solar geometry.

The ASG base lies fixed in a north-south orientation for each sample. The plane that bisects the frame symmetrically lies coincident with the north-south plane. Sampling begins with a real-time determination of the solar azimuth angle. A graphical user interface (GUI) for the motion control software on the autonomous laptop calls subroutines for solar ephemeris (given input date, time, and geographic location) and for the axis rotations (Fig. 10). The solar ephemeris subroutines (FORTRAN77) come from the Naval Research Laboratory ftp site and we modified them with FORTRAN90 calling functions.

We coded the inversion algorithms (Sec. III) for motor rotations in FORTRAN90. The ASG collects spectra from a
near-perfect reflecting white panel. The ASG arm then unfolds to full extension (θ_r = 80°, φ_r = 0°) and rotates about −z until u is in the solar principal plane, where the target, sun, and fore optic are coplanar in a vertical plane. A complete sampling then follows. The sampling path fixes view zenith angle and passes through all view azimuth angles, steps to the next smaller view zenith and repeats through to the nadir (vertical) view.

The threaded fore-optic mount at the end of the lower arm can accommodate any of the standard ASD fore optics (1°, 4°, and 8° field of view). Figure 11 shows the theoretical sampling footprint of the ASG with the 4° field-of-view fore optic relative to view zenith θ_r.

V. ANGULAR CALIBRATION

The angular accuracy of the ASG was evaluated through a laser pointing experiment. We attached the fiber optic cable of a laser to the optic mount of the ASG. In this configuration, the laser could point with each view vector u. We centered two grids with cell spacing of 0.375 cm under the vertical axis −z at the base plane of the ASG and height 1.15 cm above the base plane of the ASG. The ASG was then run through a complete 10° protocol, noting the Cartesian coordinates of the interception points of the laser with both grids. We then solved for the view zenith and azimuth of each view vector with the following relationships:

\[
\begin{align*}
\theta_r &= \cos^{-1}\left(\frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + h^2}}\right), \\
\phi_r &= \tan^{-1}\left(\frac{x}{y}\right).
\end{align*}
\]

The results of this experiment are shown in Fig. 12. The root mean squared error for zenith and azimuth were 1.3° and 2.2°, respectively, and the mean errors were 0.7° and −0.1°, respectively.

VI. HDRF COMPUTATION AND DATA FORMATS

A. Calculation of HDRF

Measuring HDRF with the ASD-FR consists of sampling spectra from a calibrated, near-Lambertian Spectralon panel.
and sampling spectra from the snow target. The HDRF for the geometry \((\theta_0, \phi_0; \theta_r, \phi_r)\) is given by

\[
\text{HDRF}_\lambda(\theta_0, \phi_0; \theta_r, \phi_r) = \left( \frac{DN_\lambda(\theta_0, \phi_0) - DC_\lambda}{C_{\lambda, \text{HDRF, Spectralon}}(0,0) - DC_\lambda} \right).
\]

\(DN_\lambda(\theta_0, \phi_0)\) is the spectral digital number recorded by the ASD-FR for the view zenith \(\theta_0\) and view azimuth \(\phi_0\) from the snow surface, \(DC_\lambda\) is the spectral dark current measurement, \(DN_{\lambda, \text{Spectralon}}(0,0)\) is the spectral digital number recorded by the ASD-FR for the nadir view \(\theta_0 = 0^\circ\), \(\phi_0 = 0^\circ\) from the Spectralon panel, \(C_{\lambda, \text{HDRF, Spectralon}}\) is the spectral correction factor for the hemispherical-directional reflectance of the Spectralon panel, and \(C_{\lambda, \text{Spectralon}}\) is the spectral calibration coefficient provided by the manufacturer that accounts for subunity directional-hemispherical reflectance of the Spectralon panel. Dark current is the digital number recorded by the ASD-FR when the fore optic is completely occulted and represents the inherent instrument noise.

The Spectralon calibration spectrum, \(C_{\lambda, \text{Spectralon}}\), is provided by the manufacturer and represents directional-hemispherical reflectance of the panel for an 8° illumination zenith angle, lying near 0.98–0.99 across most of the spectrum. Dark current is the digital number recorded by the ASD-FR when the fore optic is completely occulted and represents the inherent instrument noise.

B. Data formats

ASD-FR files are stored as 2151-band floating-point binary files with a 484 byte data header that describes instrument dynamic range, time of acquisition, and instrument configuration. With a suite of Interactive Data Language programs, we calculate the HDRF for the complete spectral \((0.35 \leq \lambda \leq 2.5 \, \mu\text{m})\) and geometric space \((0^\circ \leq \theta \leq 80^\circ\), \(0^\circ \leq \phi \leq 180^\circ\)) for each ASG acquisition. Each ASG HDRF set is stored as an ENVI spectral library, a format used in the ENVI image processing software (Research Systems, Inc., Boulder, CO). The ENVI spectral library format is a floating-point image with the number of samples equal to the number of spectral bands and the number of lines equal to the number of angular spectra. Each ENVI spectral library file has an associated header file that contains information such as data type, instrument configuration, wavelengths, and viewing geometry.

VII. RESULTS

The respective snow HDRF spectra and angular distributions of spectral HDRF at distinct wavelengths are shown in Figs. 14 and 15, respectively. These data are the best available for snow hemispherical-directional reflectance because
of the speed of acquisition, the fine angular resolution, and
the spectral range and resolution of the spectrometer.

The HDRF of snow is largest in the forward reflectance
direction ($\phi_r = 0^\circ$), increasing with view zenith angle $\theta_v$.
(Fig. 14) due to strong forward single scattering by snow
grains. The relative increase in HDRF with $\theta_v$ increases with
wavelength. Figure 15 shows the angular structure of the
HDRF at distinct wavelengths. The HDRF structure changed
from convex about the forward reflectance direction at
shorter wavelengths ($\lambda = 0.55 \mu m$) to concave about the for-
ward direction at longer wavelengths ($\lambda = 2.25 \mu m$). This
change with wavelength is due to the change in the reflect-
ance properties of snow from multiple scattering at shorter
wavelengths to single scattering (in turn due to the several
order of magnitude increase in the imaginary part of the
complex refractive index) and the structure of the single-
scattering phase function of the snow grains (which depends
on the shape and size distribution of the snow particles).\(^1\)

Measurements of the HDRF with the ASG are accompa-
nied by measurements of the atmospheric optical properties
from a sun photometer (optical depths of Rayleigh scattering,
oxygen, aerosols, and water vapor) and measurements of the
snow physical properties (grain size, grain morphology,
snow liquid water content, temperature, snow depth, and
snow density). The HDRF measurements represent the re-
lected radiance field at the bottom of the atmosphere,
whereas a satellite measures one direction in this radiance
field convolved with transmission through and scattering and

FIG. 14. Snow HDRF spectra (left) collected with the ASG with $\theta_v = 47^\circ$ at
10° angular resolution. The top plots correspond to a view azimuth of $\phi_r =
0^\circ$, the middle plots correspond to a view azimuth of $90^\circ$, and the bottom
plots correspond to a view azimuth of $180^\circ$. The surface grain size, esti-
mated with stereological analysis, was $\sim 240 \mu m$.

FIG. 15. (Color) Polar plots of the snow HDRF shown in Fig. 14 for all
view angles for six wavelengths across the solar spectrum. The radial dis-
tance from the center of each plot represents the view zenith angle. Rotation
about the center represents a change in azimuth. The azimuth angle of $0^\circ$ is
the forward reflectance half of the solar principal plane and illumination
comes from an azimuth angle of $180^\circ$.\(^1\)
absorption by the atmosphere. Therefore, the ASG HDRF measurements are used as a validation or boundary condition for an atmospheric transmission model coupled with a remote sensing inversion model. Laboratory measurements of directional reflectance lack a diffuse irradiance component (that is, they are truly bidirectional reflectance measurements) and are therefore one step removed from the radiance field sampled by the satellite. It should be noted though that in alpine terrain, atmospheric optical depths are small, and therefore diffuse irradiance decreases with increasing wavelength. At wavelengths greater than \( \sim 1.0 \, \mu m \), the HDRF and bidirectional reflectance factor are nearly identical.\(^7\)

Coupling these data with measurements of snow properties in the field facilitates improvements to radiative transfer models of snow optical properties and ultimately models that invert remotely sensed data for spatial distributions of snow properties. For example, Leroux \textit{et al.}\(^6\) made measurements of the directional reflectance of snow in the near infrared. They demonstrated that results from their radiative transfer model\(^26\) better matched the directional reflectance measurements if hexagonal particles were used for single-scattering calculations rather than spheres. Furthermore, Painter \textit{et al.}\(^27\) used a similar radiative transfer model to generate a spectral library of snow spectral directional reflectance to invert imaging spectrometer data for fractional snow cover, grain size, and albedo in alpine drainage basins. The improvements in physical property retrievals provided by the incorporation of knowledge of the directional reflectance of snow will ultimately improve snow hydrologic modeling in rough terrain.\(^4,28\)

The scale of the ASG permits application to other smooth surfaces such as desert, soil, tundra, and pavement. The authors are currently generating a database of high angular and spectral resolution HDRF measurements with the ASG. The data can also be made available to incorporation into databases such as the Columbia-Utrecht Reflectance and Texture Database\(^29\) and that being provided with a coming text on field measurements of directional reflectance.\(^30\)

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