A Method for Satellite Identification of Surface Temperature Fields of Subpixel Resolution

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It is possible to measure surface radiant temperature fields of subpixel spatial resolution from satellites which contain more than one channel in the thermal infrared spectral region. Because of the different response of the Planck function at different wavelengths, the radiant temperatures measured in two channels may be expressed in terms of contributions from two temperature fields, each occupying a portion of the pixel, where the portions are not necessarily contiguous. The resulting simultaneous nonlinear equations may be solved for the complimentary portions of the pixel occupied and one unknown temperature. In two adjacent pixels which can be assumed to have the same target temperatures and same background temperatures, both unknown temperatures may be found.

1. Introduction

Satellite spaceborne radiometers with more than one thermal infrared channel were originally designed to identify cloud-contaminated pixels and to evaluate the radiant contribution from atmospheric water vapor. However, over a surface with markedly varying temperatures at subpixel resolution, the different radiant temperatures sensed at different wavelengths may be more a result of the surface temperature field than of atmospheric interference with the signal. In this paper, a method is presented for identifying the magnitudes and the subpixel areal coverages of two different surface radiant temperatures from TIROS-N series satellite data.

Over a land surface with two different surface materials of different temperatures, the way in which the radiance contributions are averaged over a pixel will depend on the wavelength range of the sensor. If one part of a pixel is much warmer than the remainder, for example, that warm part will contribute proportionally more radiance to the signal in shorter wavelengths in the thermal infrared than in longer wavelengths. Through manipulation of the integral of the Planck function for the different channels, it turns out to be possible to calculate (a) the radiant temperatures of one of two temperature fields of subpixel resolution, and (b) the portions of the pixel that each occupies. The portions of a pixel occupied by each temperature field are not necessarily contiguous, but the method does assume that there are only two temperature fields, with a "target" temperature and a "background" temperature.

2. TIROS-N Satellite Series Thermal Infrared Sensors

NOAA's TIROS-N series polar orbiting satellites now contain instrumentation to measure upwelling thermal infrared radiation in more than one wavelength band.
TABLE 1 AVHRR Thermal Channels (from Schwalb, 1979)

<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>WAVELENGHT RANGE (µm)</th>
<th>NOAA-6</th>
<th>WAVELENGHT RANGE (µm)</th>
<th>NOAA-7 (and subsequent satellites)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>3.55 - 3.93</td>
<td>3</td>
<td>3.55 - 3.93</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10.5 - 11.5</td>
<td>4</td>
<td>10.3 - 11.3</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>11.5 - 12.5</td>
<td>5</td>
<td>11.5 - 12.5</td>
<td></td>
</tr>
</tbody>
</table>

The Advanced Very High Resolution Radiometer (AVHRR) on NOAA-6 (launched June 1979), with a spatial resolution of 1.1 km, has two thermal infrared channels. On later satellites in the series (beginning with NOAA-7 to be launched in the late summer of 1981) a third thermal channel will be added to the AVHRR. Wavelength ranges for these instruments are given in Table 1.

Channels 3 and 4 on these satellites are in water vapor window regions of the electromagnetic spectrum. The design purpose of the two channels was to allow discrimination of cloud-contaminated pixels, and to provide a measurement of upwelling radiance in a highly transmissive water vapor window, for the measurement of sea surface temperature, chiefly at night, because of the problem of contamination by reflected solar radiation in channel 3. The third thermal infrared channel will add capability to determine absolute sea surface temperature during day or night.

3. Notation

- \( c_1 \) First Planck constant (3.741832 \( \times 10^{-16} \) W m\(^{-2}\))
- \( c_2 \) Second Planck constant (1.438786 \( \times 10^{-2} \) m K)
- \( L(T) \) Upwelling thermal radiance (W m\(^{-2}\) \( \mu m^{-1} \) sr\(^{-1}\)) as a function of temperature
- \( L_j(T) \) Integrated upwelling thermal radiance in channel \( j \) (W m\(^{-2}\) \( \mu m^{-1} \) sr\(^{-1}\))
- \( p \) Portion of a pixel occupied by target
- \( T \) Temperature (K)
- \( T_i \) Radiant temperature measured by channel \( i \) (K)
- \( T_b \) Background temperature (K)
- \( T_t \) Target temperature (K)
- \( \beta(\lambda, T) \) Planck function (W m\(^{-3}\))
- \( \epsilon \) Emissivity
- \( \lambda \) Wavelength (\( \mu m \) or m)
- \( \Phi(\lambda) \) Spectral response function of sensor

4. Radiance-Temperature Relations

In the absence of an atmospheric contribution or attenuation, the upwelling radiance sensed by a downward-pointing radiometer is derived by integrating the product of Planck's function and the response function of the sensor,

\[
L(T) = \frac{\frac{1}{\pi} \int_0^\infty \epsilon_\lambda \beta(\lambda, T) \Phi(\lambda) d\lambda}{\int_0^\infty \Phi(\lambda) d\lambda} \quad (1)
\]

For most earth surfaces, \( \epsilon_\lambda \) is relatively independent of \( \lambda \) over the range of an AVHRR channel, so one can drop the \( \lambda \) subscript and move \( \epsilon \) outside the integral in (1) The Planck function for emittance at wavelength \( \lambda \) of a blackbody at temperature \( T \) is (Suits, 1975)

\[
\beta(\lambda, T) = \frac{c_1 \lambda^{-5}}{\exp(c_2/\lambda T) - 1} \quad (2)
\]
The relative spectral response function for the sensor \( \Phi(\lambda) \) is zero outside some range \([\lambda_1, \lambda_2]\). For rough calculations it is often defined as a "gate function," i.e., equal to one within the range of values specified in Table 1. Alternatively, it may be defined more precisely as a graphical or tabulated function, as Fig. 1 shows for channels 3 and 4 of NOAA-6.

Let us designate \( L_3(T) \) and \( L_4(T) \) to be the NOAA-6 channel 3 and 4 radiances as functions of blackbody temperature (i.e., \( \varepsilon = 1 \)), and the inverse functions by \( L_3^{-1}(\cdot) \) and \( L_4^{-1}(\cdot) \), respectively. For temperatures from 100–1000 K, Eq (1) was calculated numerically by an adaptive quadrature method (Forsythe et al., 1977, chapter 5). The resulting values for
$L_3(T)$ and $L_4(T)$ are shown in Fig. 2. Here it is apparent that the shift in the peak of the Planck function toward shorter wavelengths, as temperature increases, causes the radiant contribution to channel 3 to increase more rapidly, at higher temperatures, than that to channel 4.

Now, suppose we have a “mixed pixel” composed of a target at temperature $T_t$ which occupies portion $p$ of the pixel (where $0 \leq p \leq 1$) and a background temperature $T_b$ which occupies the remaining portion $(1-p)$ of the pixel. The radiant temperatures sensed by the AVHRR in channels 3 and 4 will be, in the absence of an atmospheric contribution or attenuation,

$$T_j = L_j^{-1}[pL_j(T_t) + (1-p)L_j(T_b)],$$

where $j=3,4$ (3)

![Figure 3](image1.png)  
**FIGURE 3**  $T_3$ (top) and $T_4$ (bottom) vs $p$ for increasing values of $T_t$, for $T_b=285$ K. The dashed line in the lower graph shows the range of values permissible when $T_3=325$.  

![Figure 3](image2.png)
For a reference background temperature $T_b = 285$, Fig. 3 shows values of $T_3$ and $T_4$ as functions of $p$ for various values of $T_t$ from 150 to 500 K. In Fig. 4 are plotted the differences $T_3 - T_4$ versus $p$ for the same situations.

5. Determination of $T_t$ and $p$

Assume that we know the value of $T_b$, perhaps from measurements from surrounding pixels which contain no "target" surface. The two equations in (3) now have only two unknowns, $p$ and $T_t$. From Fig. 3, it is evident that we can solve for them. For example, suppose $T_b = 285$, $T_3 = 325$, and $T_4 = 307$. The value of $T_3$ constrains the range of choices considerably, as shown by the dashed line in the lower graph in Fig. 3. The correct values for $p$ (0.2) and $T_t$ (371) can be selected from the value of $T_4$. Similarly, the correct value can be read from the difference $T_3 - T_4$ (18) in Fig. 4. Note that the largest differences between $T_3$ and $T_4$ occur when the pixel contains a small, hot target, or when a cold target covers almost all of the field of view. If the difference between $T_t$ and $T_b$ is not very large, the solution is insensitive to variations in $p$.

A mathematical solution which does not require the construction of graphs for every value of $T_b$ is accomplished by rearranging (3) in these simultaneous non-linear equations.

$$L_i(T_i) - [pL_i(T_t) + (1-p)L_i(T_b)] = 0,$$

$$i = 3, 4$$

There are several methods of solving such sets of equations. I use a modification of Brent's (1973b) algorithm developed by Moré and Cosnard (1979). Convergence problems are reduced if Box's (1966) transformation is used to map the constrained temperatures into unconstrained variables.

6. Determination of $T_t$ When $T_b$ and $p$ Are Unknown

For some applications, for example determination of sea surface temperature,
we have no reliable estimate of the background temperature $T_b$. However, suppose we assume that in two adjacent pixels the target (sea surface) temperatures $T_t$ are the same and the background (cloud) temperatures $T_b$ are the same, but that the cloud-covered portions $(1 - p)$ are different.

Note that Eq. (4) can be solved for $p$,

$$p = \frac{L_i(T_t) - L_i(T_b)}{L_i(T_t) - L_i(T_b)} , \quad j = 3, 4. \quad (5)$$

If we denote the non-cloud-contaminated portions of two adjacent pixels as $p^{(1)}$ and $p^{(2)}$, and their radiant temperatures by $T^{(1)}$ and $T^{(2)}$, we can form the ratio

$$\frac{p^{(1)}}{p^{(2)}} = \frac{L_i(T^{(1)}) - L_i(T_b)}{L_i(T^{(2)}) - L_i(T_b)} , \quad j = 3, 4, \quad p^{(1)} \neq p^{(2)} \neq 0. \quad (6)$$

Similarly, if Eq. (4) is solved for $(1 - p)$ instead of $p$, we arrive at:

$$1 - p^{(1)} = \frac{L_i(T^{(1)}) - L_i(T_t)}{L_i(T^{(2)}) - L_i(T_t)} , \quad (7)$$

Although the values of the ratios in Eqs. (6) and (7) are unknown, Smith and Rao (1972) have pointed out that they are independent of wavelength and therefore must be the same for all values of $j$. Hence

$$\frac{L_3(T^{(1)}) - L_3(T_x)}{L_3(T^{(2)}) - L_3(T_x)} = \frac{L_4(T^{(1)}) - L_4(T_x)}{L_4(T^{(2)}) - L_4(T_x)} = 0,$$

$$T^{(1)} \neq T^{(2)}; \quad T^{(1)} \neq T^{(2)} \quad (8)$$

In Eq. (8) the variable $T_x$ designates some unknown temperature $T_x$.

Examination

![Graph of Eq (8)](image-url)

**FIGURE 5**  Graph of Eq (8) for $T^{(1)} = 261.4$, $T^{(2)} = 274.6$, $T^{(1)} = 241.5$, and $T^{(2)} = 262.9$

The solutions are $T_b = 210$ and $T_t = 285$
of the equation (see Fig. 5) indicates that it has two roots \((T_b, T_t)\) and two discontinuities \((T_i^{(2)}, T_k^{(2)})\). It is possible to solve Eq. (8) by an iterative method for \(T_b, T_t\), but the method and the starting guesses must be chosen carefully. My recommendation is to use Brent’s (1973a, chapter 4) algorithm. It requires two initial guesses which span the solution, but problems with the discontinuities can be avoided by insuring that the guesses do not span either discontinuity.

Once \(T_b\) is found, it can be used in Eq (5) to find \(p\).

7. Corrections for Atmospheric Effects

Because some of the thermal radiation emitted by the earth’s surface is absorbed and re-emitted by the atmosphere, radiant temperatures measured from space are generally lower than the true surface radiant temperature, although they can be higher where there are warm, moist inversion layers. Dual thermal channels, in different wavelength bands, can be used to correct for atmospheric attenuation and thereby determine the absolute surface temperature, to within about 1 K. Prabhakara et al. (1974) and Morcrette and Irbe (1978) have shown theoretically that a correction of the following form gives the actual surface radiant temperature, for a variety of possible atmospheric temperature and water vapor profiles

\[
T_{\text{surf}} - T_i = a(T_i - T_k) + b
\]

\(T_i\) and \(T_k\) are satellite radiant temperature measurements in two separate channels. The coefficients \(a\) and \(b\) depend on the wavelength values of the channels. McClain and Abel (1977) have verified the method using coincident satellite and ship data sets. For channels 3 and 4 of NOAA-6, McClain (1980), using the atmospheric radiance model of Wemreb and Hill (1980), has calculated the coefficients to be \(a = 0.42\) and \(b = 1.3\), when \(j = 3\) and \(k = 4\).

To determine the necessary atmospheric correction when \(T_b\) is known, one need only determine the magnitude of the correction from the pixels surrounding the one containing the target temperature field. From these the additive correction to \(T_3\) and \(T_4\) can be made, and the analysis can proceed as described in section 5.

The case where \(T_b\) and \(p\) are unknown is somewhat more complicated, but can be solved if there are three, instead of two, pixels where \(T_b\) is the same and \(T_i\) is the same, but where \(p^{(1)} \neq p^{(2)} \neq p^{(3)}\).

Let us introduce the notation \(T_{b,3}\) to designate the brightness temperature that the satellite would register if the entire pixel were of the background temperature, i.e., \(p = 0\). Let \(T_{i,3}\) designate the brightness temperature if the entire pixel were of the target temperature, i.e., \(p = 1\). \(T_{b,4}\) and \(T_{i,4}\) are similarly defined. If we form the ratio

\[
\frac{1-p^{(1)}}{1-p^{(2)}}
\]

we can derive an equation identical to (8) but with two unknowns, \(T_{x,3}\) and \(T_{x,4}\) instead of one. To solve for these we need an additional equation, which we form from the ratio

\[
\frac{1-p^{(3)}}{1-p^{(2)}}
\]
and thereby convert (8) into two simultaneous nonlinear equations.

\[
\frac{L_3(T_3^{(k)}) - L_3(T_{x,3})}{L_3(T_{3}^{(2)}) - L_3(T_{x,3})} - \frac{L_4(T_4^{(k)}) - L_4(T_{x,4})}{L_4(T_4^{(2)}) - L_4(T_{x,4})} = 0, \quad k = 1,3,
\]

\[T_3^{(1)} \neq T_3^{(2)} \neq T_3^{(3)}, \quad T_4^{(1)} \neq T_4^{(2)} \neq T_4^{(3)}.\]

In Eq. (10) the variables \(T_{x,3}\) and \(T_{x,4}\) designate the roots. There are two sets of solutions \((T_{b,3}, T_{b,4})\) and \((T_{r,3}, T_{r,4})\). Once these are determined, \(T_b\) and \(T_r\) can be found from Eq (9).

8. Discussion

This algorithm makes possible (under some circumstances) the determination of temperature fields of subpixel resolution. Potential applications include the following.

1. It might be possible to determine temperatures of urban heat sources. Tests are being conducted within NOAA to estimate stack temperatures of steel plants, using NOAA-6 satellite data with 1-km resolution.

2. The technique might be useful in estimating surface temperatures and areal extents of geothermal areas.

3. In areas that are partly snow covered, the method could be used to estimate snow areal extent in each pixel.

4. In areas where there are no cloud-free pixels, but where there are some pixels which are not totally cloud-covered, the method still provides a measurement of sea-surface temperature, including corrections for atmospheric attenuation due to water vapor.

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