

Learning, Irreversibilities and Climate

by

Charles D. Kolstad¹
Department of Economics
University of California
Santa Barbara, CA 93106-9210 USA

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I. INTRODUCTION

Climate change is a currently visible example of a common type environmental problem, characterized by its stock nature: pollution slowly accumulates over time with the stock of pollution causing the damage. This is also the case with acidification of lakes due to acid rain, accumulation of hazardous wastes, groundwater contamination and stratospheric ozone depletion. An important characteristic of stock pollution problems is that pollution typically accumulates over long periods of time resulting in a stock that is difficult to eliminate or reduce if a time comes when that is socially desirable or imperative. Anthropogenic greenhouse gases have been accumulating for over two hundred years; even if we were completely convinced that the levels of these gases are too high, there is little we can do that will appreciably reduce greenhouse gas levels within the time frame of decades or even centuries.

A key aspect of most of these environmental problems is a high degree of uncertainty-- uncertainty about the physical processes involved, uncertainty about the physical or biological damages to people and ecosystems, not to mention the value of those damages, and uncertainty about the costs of controlling the problem. This makes the problem complicated enough to make determining optimal pollution control strategies difficult; however, the fact that our information is changing as we learn about the problem creates another layer of complexity. Uncertainty by itself usually calls for acting on the basis of expected utility or perhaps expected costs and benefits. However, when the underlying probability distributions are changing over time in unpredictable ways, the decision problem becomes more complex. Should an otherwise optimal action be delayed because of the expected arrival of information? Should we be investing in acquiring

information rather than pollution control?

To an analyst, dealing with an uncertain world is more difficult than a deterministic world--distributions as well as expected values of parameters must be estimated and computation becomes more complex. However, dealing with learning makes the task even more difficult. Not only must uncertainty be represented but the mechanism whereby the probability distributions change over time must also be represented. Furthermore, if learning is fully endogenized within a model of climate change, then the production of information must also be represented.

There are two purposes of this paper. One is to add some structure to thinking about uncertainty and learning in the context of analysis of climate change. The second is to review approaches for including learning within analytic models of climate and the economy ("integrated assessment" models). These purposes are taken up in turn in each of the next two sections of the paper.

II. THE POLICY DIMENSIONS OF UNCERTAINTY AND LEARNING

The climate and the economy can be represented quite simply as a dynamic system. At the simplest level, state variables are the stock of greenhouse gases, the stock of capital, the level of atmospheric temperature, and the stock of information. All of these states evolve over time and can be controlled by investments in capital accumulation, emission control and information acquisition. In discrete time, this can be most generally written as the following infinite horizon control problem:

$$\begin{aligned} \max_c \quad & \bar{\omega} \sum_{t=0}^4 \beta^t U(s_t, c_t, a, g_t) & (1a) \\ \text{s.t.} \quad & s_{t+1} = g(s_t, c_t, \beta, \epsilon_t) & (1b) \end{aligned}$$

In equation (1), the problem is to maximize the expected net present value of utility using controls, c_t , where the discount factor is β . Utility is a function of the states, s , controls, c , a parameter vector, a , and a stochastic shock, ϵ_t . The state vector evolves according to the function g , which depends on the states and controls as well as another parameter vector, β , and another stochastic shock, ϵ_t . This is a very general representation of the climate process and encompasses nearly all equilibrium integrated assessment models.¹

A. Uncertainty vs. Stochasticity

There are two types of randomness that enter into model (1) -- uncertainty and stochasticity. As we use the term here, by *uncertainty* we mean aspects of the problem that are deterministic but simply not known to the analyst or to the "control problem." For instance, a in eqn. (1) may have a specific value but that value may be unknown; in this case we characterize a by a probability distribution. What is uncertain is our knowledge of the specific value of a . The same applies to β . Contrast this to the cases of ϵ_t and ϵ_t , which every period take different values,

¹Most models do not incorporate the stochastic elements of this general model, nor do they involve truly infinite horizons. The standard reference for equilibrium integrated assessment models is Nordhaus (1994) which is a Ramsey-type optimal growth model with climate states being the level of greenhouse gases, the level of atmospheric temperature and the level of the deep ocean temperature; the only economic state is the capital stock and controls are investment and emission control rates. He limits his horizon to several centuries.

drawn from an underlying distribution. Think of these as shocks to the system or sources of *stochasticity*.

Although both types of parameters are random variables, there is a key difference. Parameters that are uncertain are potentially knowable, if enough effort is expended to ascertain their true value. Stochastic shocks take on different values every time period; there is typically little value in learning the true level of a past shock, particularly if we assume general knowledge of the distribution from which the shock is drawn. An example of an uncertain parameter in the case of climate change is the climate sensitivity. There is considerable debate over exactly how much the global mean temperature can be expected to rise from a doubling in carbon dioxide. This climate sensitivity is the subject of much debate. Some view it to be less than 1°C while some view it to be much higher.² But whatever the value, it does not change over time; thus research as well as a longer climate record will narrow the uncertainty over time.³

A stochastic variable, however, is not amenable to learning or information acquisition. For instance, the average weather (eg, mean temperature) over a year will vary from one year to the next in an unpredictable fashion (despite what weather forecasters may say). Or the amount of technical progress in an economy or sector of an economy will vary from year to year in an unpredictable fashion.

Stochasticity is important for two reasons. One reason is that a stochastic model will

²In a survey of scientists on their opinions of the value of climate sensitivity to a doubling of carbon dioxide, estimates were concentrated in the 1 to 4 degree range (Morgan and Keith, 1995).

³It is important to realize that learning or gaining information about a random variable need not reduce the variance of the random variable. For instance, if one knows that a variable is normally distributed with a mean of zero and a variance of one and then learns that the variable is greater than two, the variance of the posterior will be quite high; yet information has unequivocally been gained.

always have uncertainty and unpredictability. Secondly, stochasticity prevents the rapid learning about an uncertain but non-stochastic variable. For instance, if we could precisely measure the level of greenhouse gases and the global mean temperature, and there were no shocks to the system governing the evolution of climate, we would very rapidly be able to infer the values of the uncertain parameters governing climate evolution. This is more than an idle example. Figure 1 shows the global instrumental temperature record, going as far back in time as it is available. A considerable amount of the debate on the physical dimensions of climate change involves inferring the value of the climate sensitivity parameter from this temperature record, and the constructed record of greenhouse gas levels (also shown in Figure 1). It is obviously the stochastic element that prevents more certain inference regarding climate sensitivity.

Another example where stochasticity clouds underlying information can be found in climate damages. One of the real ways of measuring the damage from climate change will be to measure the effects of climate change in the field. For instance, measuring the loss in output in agriculture associated with elevated temperatures or other statistics of climate will involve separating the effect on productivity (or output) from factors such as fluctuations in factor prices and technical progress from the effect of climate. Figure 2 shows output per acre of corn in Iowa in the US for a forty year period. Without the stochasticity it would be much easier to read the effect of climate on output; with stochasticity, any signal regarding damage is masked.

B. Risk

A second issue closely related to uncertainty is risk. In the context of climate change, an underlying theme is that the potential dangers of upsetting the earth's climate are so great that we

should be cautious, avoiding risky activities. Some regulatory strategies for controlling greenhouse gas emissions are touted based on the fact that they provide insurance against unknown but potentially severe climate effects. Another argument made for such policies is that we must get the greenhouse gas genie back in the bottle before it is too late to do so.

There are really two issues here. One is risk aversion. If the range of possible outcomes is sufficiently large, then there may be a substantial value associated with reducing the variability in the outcomes. This is the sense in which an a precautionary action can have "insurance value." Doing nothing to control climate change may result in large damage with low but positive probability and no damage with some positive probability less than one. Taking a modest amount of costly precautionary action now may reduce the outliers in both directions, effectively reducing the variance in outcomes. This is not insurance in the sense of pooling multiple independent risks, which is the idea behind typical insurance markets, where risk can in effect be costlessly reduced.⁴

It may help to offer a specific example. Suppose as a consequence of today's emissions, the world could end up in two possible states in a century's time -- high temperature change/damage (H) or low temperature change/damage (L). Control costs are incurred today.

To appreciate the insurance value of an action, Figure 3 shows a hypothetical utility function for two levels of change in the global temperature. Little loss would be associated with small temperature changes and much greater loss with larger temperature changes. Shown are two possible states of the world in the absence of intervention: L_1 and H_1 . Also shown is the expected temperature/damage, $ED = (1-p) L_1 + p H_1$ where p is the probability of H_1 being

⁴Chichilnisky and Heal (1993) propose an interesting market for contracts whose payoff depends on the realized climate: climate-change optimists sell to climate-change pessimists.

realized. The expected utility is shown as U_1 . Now suppose we are considering some action that will reduce the size of the extremely high temperature event but also increase the size of the low temperature outcome, in such a way that the expected temperature stays the same. Thus we are decreasing the variance in possible temperature outcomes but keeping the mean the same. These outcomes are shown as (L_2, H_2) . The resulting expected utility is U_2 . The value of this reduction in the variance of the outcomes, is the monetary value of the utility change, $(U_2 - U_1)$. The expected climate change remains at ED, yet the regulatory action has an "insurance" value.⁵

For there to be an insurance value associated with a regulatory action to control climate change, two conditions must hold: society must be risk averse and the magnitude of the potential damage must be very high. It is well known that even though citizens may be risk averse, society is usually risk neutral towards most public actions,⁶ simply because such actions (and their potential up-sides and down-sides) are usually small relative to the assets of society. Thus social risk aversion, if it exists, will be much less than individual risk aversion. The second criterion that must be met (not unrelated) is that the potential damage must be very large. If the damage is not very large, we are back at the issue of whether society is risk averse with respect to modest risks.

Are the potential damages from climate change large or modest, relative to other social risks? This is not an easy question to answer although we should be careful in not mistaking a large effect for a large effect in the context of other risks society faces. In other words, losses of several tens of billions of dollars a year in Europe or North America are certainly large but amount to less than 1% of gross output. This is not to say that we should not care about such

⁵See Manne and Richels (1992).

⁶See Arrow and Lind (1970).

risks; rather to say that decisions should possibly be made on the basis of expected value of damage without a risk premium.

One of the most severe events that could occur in the Northern hemisphere is a shut-down of the deep circulation in the North Atlantic. This would make the climate of Europe more akin to northern Norway. Is this a catastrophe? This would have major negative consequences for much of Europe. For instance, Europe's agriculture would clearly suffer; but most industries are relatively immune to climate change and would be more modestly affected. It is certainly an effect to be avoided but is there a risk premium and if so, how large is it?

B. Irreversibilities

Another closely related issue but distinct is that of irreversibilities. In the context of the shut-down of the North Atlantic current mentioned above, if a reduction in greenhouse gas levels can restore the current, then the issue is one of the magnitude of damages, not whether we will get into a corner from which there is no exit.

Irreversibilities are only significant when learning is taking place. This is important and not always appreciated. Let us use our example where climate change can lead to two levels of damage--high and low. The damage will occur in the future while the control costs will occur in the present. The question is, how to decide on the proper level of control today? Suppose the high damage case discussed in the context of Figure 3 is associated with an irreversible change in the climate (such as the triggering of an ice age in Northern latitudes). How do we take that possibility into account? Clearly there is high damage associated with such a catastrophic event. The standard decision rule for deciding on control actions to take today, is to choose controls that

maximize the expected value of utility. Does the fact that one of the possible outcomes is irreversible play any role here? Only if we expect more information to be forthcoming. If we expect to know more tomorrow, then it is optimal to delay actions that could lead to irreversibilities. If we do not expect to know anymore tomorrow, then our actions will not change tomorrow. Thus we will have no cause tomorrow to regret today's decision.

Some would suggest that we should avoid any action that has a chance, however slight, of causing catastrophic consequences. An examination of more familiar decision-making will illustrate the fallacy here. We all take risks in our everyday activities, and the down-side of those risks are often catastrophic. Auto travel can lead to very irreversible and catastrophic outcomes for us as individuals. Yet that does not mean we eschew auto travel. The disutility of dying is great but not minus infinity.⁷ We are careful but willing to take risks.

Now suppose we introduce some learning. Suppose we are contemplating renting a car and know that the probability of catastrophic failure of the car is 10^{-6} . If the gain from renting the car is great enough, most of us will probably take the risk. Now suppose we are told that tomorrow a new test will be available to identify the cars prone to failure and that driving tomorrow can be risk free. Will we change our behavior? Many of us would decide to forgo our activities for today and wait until tomorrow to rent the car. The *fact* that information will be acquired influences our actions today. Without learning, death is simply a consequence with very high disutility; its irreversibility is not fundamental in making our decision about whether or not to rent a car.

⁷In her paper on environmental catastrophes, Cropper (1976) defines a catastrophe as an event equivalent to reducing consumption to zero and thus utility to zero (not negative infinity).

An alternative way of viewing this is in terms of option value. There is an option value of retaining flexibility--keeping options open. But if learning never takes place, there will never be a need to exercise an option and thus no option value. Alternatively, by taking an irreversible action, the value of learning is degraded. Thus there is an option value associated with not taking an irreversible action.⁸

The implications for climate change policy are simple. We should take seriously the possibility of catastrophe. But the remote possibility of catastrophe should not trump actions today. On the other hand, if we are learning over time about the likelihood of catastrophe and our actions today could have an affect on the probability of catastrophe, then we should probably err on the safe side, at least somewhat, while we are acquiring information. How much precaution to take is an empirical question.

Another point should be made here and that is that precaution due to irreversibilities and learning is not the same as the insurance effect. The insurance effect, described previously, requires risk aversion. However, precaution due to learning and irreversibilities may be warranted for risk-neutral or even risk-loving individuals. The main factor driving this result is learning or differences in the amount of available information over time.

In the context of climate change, it is important to keep in perspective the time constants involved in learning and in climate change. Consider the optimal time-paths of carbon emissions published by Nordhaus (1994) in his book. Figure 4 shows his calculated optimal path of carbon

⁸There is a very large literature on option value in environmental decision making, not to mention the large literature in finance. See the book by Dixit and Pindyck (1994) for a review of the issues involved in irreversible investments under uncertainty, including an application to climate change.

emissions (labelled as M). While reasonable people can debate the validity of his calculated optimal trajectory, let us put that aside for the time being. Also shown are two alternate paths of emissions (not from Nordhaus), one if damages from warming are low (L) and one if damages are high (H). The increase in the global mean temperature associated with the M emissions trajectory is roughly linear, reaching 3EC in roughly 2100. Thus warming is a slow process.

Now where is the irreversibility? One possible irreversibility is the triggering of a catastrophic environmental event. Another possible irreversibility is that at some point in the future we may find damage from climate change to be very high (though not catastrophic) and may wish to reduce levels of greenhouse gases in the atmosphere. We can potentially reduce emissions to zero but we will have a hard time reducing emissions below zero (in essence, sucking carbon out of the atmosphere).⁹ So there is an irreversibility on the emissions side. A third irreversibility is associated with pollution control capital. Although there are many ways of controlling carbon emissions, one way is through the use of abatement capital. If at some point in the future we find out that climate change is a non-issue, we may wish to reduce levels of abatement capital but will be unable to do so rapidly.

The first two of these irreversibilities would call for erring on the side of the environment, emitting less than might be optimal without learning. But how much precaution should be taken? If in fact learning is proceeding relatively rapidly, there may be little or no irreversibility. This is illustrated in Figure 4. Suppose we expect to learn a great deal about the damage from climate change over the next two decades (uncertainty resolved at A). What is the down side from

⁹Actually, this really amounts to large costs of reversal. We can reduce the atmospheric stocks of carbon but at very high costs -- massive tree plantations for example.

emitting what may turn out to be too much (if H is revealed to be the true state of the world). If we learn that damages are high, we can correct our error by under-emitting for a decade or two until we are back on the optimal trajectory. Because of the long residence times of carbon, and the fact that the stock of carbon is what matters, over emitting for a few decades can be offset by under emitting for a few decades. In the end the stock will be the same. If all that matters is where we are at the end of the century, then clearly there is no loss from waiting until information arrives. The climate irreversibility does not bind.

If on the other hand, we don't expect to know the truth for a very long time (say point B), then in order to correct our over emitting, we would need to emit negative quantities and cannot. This is illustrated in Figure 4. This is an irreversibility that may call for some precautionary behavior in advance of resolution of uncertainty.

C. Learning and Adaptation

We have been speaking about learning as if knowledge is gained and shared equally at any point in time and that all agents in an economy use the information in a similar fashion. This is an oversimplification, particularly in the case of climate change. A simple distinction is between the regulator of carbon emissions and the economic agent within the economy.

The regulator is interested in designing a regulatory instrument to achieve the optimal level of emissions of greenhouse gases. The regulator may be uncertain about many dimensions of the problem, including costs and benefits. The question is, what is the optimal regulatory strategy in this environment of uncertainty and learning. This issue has been treated by a number of integrated assessment models of climate change.

Manne and Richels (1992) introduce the concepts of "learn then act" and "act then learn" into the climate change debate. They distinguish between the case when one must take action before uncertainty is resolved ("act then learn") from the case where uncertainty is resolved first ("learn then act"). Nordhaus (1974) treats the issue of sequential learning and the value of obtaining perfect information at different points in time. He finds very little effect of changing the rate of learning on current period control levels.

Kolstad (1994ab) considers both the emissions irreversibility and the capital irreversibility. He finds that in comparison to the emissions irreversibility, the capital irreversibility has a stronger influence on today's control decisions, resulting in somewhat less abatement than would have been the case with no learning. The basic reason is that the rate of learning is such that one is more likely to invest in capital that is subsequently deemed useless (due to discovering climate change is a non-problem) than to have to reduce emissions below zero (due to discovering climate change is a very large problem).

The regulators are of course not the only ones in an economy who are learning. Individual agents may need to know the state of the climate in order to make decisions; imperfect knowledge about the climate will lead to imperfect decisions, at least when compared to the case of perfect information. For instance, a builder deciding whether to locate a building near the ocean should have information about future sea-level rise. A farmer deciding what crops to plant needs an estimate of the weather over the coming growing season; if the climate has changed without her knowledge, she may make suboptimal decisions, at least in comparison to having complete information about the state of the climate.

To illustrate, consider Figure 5. Shown in the figure is a hypothetical relationship between

farm output and the extent to which realized weather differs from expected weather.¹⁰ The top figure shows how output might vary with no climate change ($T=0$). If the farmer correctly forecasts the weather, the ratio of actual to expected weather will be $r_0 = 1$; the expected output is Y_0 . Incorrect forecasts would lead to lower output. If the climate changes and the farmer realizes it, maximum output drops to Y_2 . If however, the climate changes and the farmer does not realize it, the ratio of the actual weather to his forecast of the weather will typically be high, say r_1 . In this case, output drops to Y_1 .

The point here is that while an economic agent that is fully informed about the climate may be able to adjust to climate change, an agent who does not realize the climate has changed may suffer considerable damage while she is learning. In fact, this could be argued to be the essence of the transition costs of moving from one climate to another. It could also be argued to be the essence of adaptation.

In summary, there are two effects of learning. At the most basic level, agents within the economy must adjust their production and consumption decisions as they learn about climate change. While they are learning, suboptimal decisions will be made and damage incurred, relative to the case of perfect information about climate change. This is adaptation. At a second level, regulators need to adjust their control of aggregate emissions. The optimal level of control depends on the level of information a regulator has about climate and climate change.

To my knowledge, no integrated assessment models of climate change deal with adaptation as has been described here. A few models deal with learning on the part of the

¹⁰The figure represents a long run relationship since it would obviously be possible in the short-term to have a "good-weather" shock.

regulator.¹¹

III. REPRESENTING LEARNING IN ECONOMIC MODELS

In the previous section we considered the implications of risk, uncertainty and learning for climate change policy. We also considered the extent to which typical integrated assessment models deal with the issues. We now turn to the question of how to represent learning within an economic model. Our ultimate goal is to include learning within an integrated assessment model of climate change.

Information and learning has been an important part of economics for decades. In fact, over the past few decades, a good deal of what had appeared to be anomalous behavior in the economy has been explained on the basis of incomplete, imperfect or asymmetric information. Learning, as developed in the literature, can be divided into three basic types: passive, active and purchased. Passive learning is exogenous learning and involves the arrival of information over time like manna from heaven; the economy or the agent within an economy has no control over the rate at which information arrives nor its quantity. Purchased information involves making the learning process more endogenous, though not completely endogenous. Information is available, but must be purchased, at a cost. For instance, research and development may be undertaken at a cost. The mechanism whereby information is generated may not be explicit; but all that counts is that information may be obtained at a cost. Active learning involves endogenizing the information generation (experimentation) process. For instance, we may be unsure about the relationship

¹¹See Kolstad, 1993, 1994b; Manne and Richels, 1992; Nordhaus, 1994; Peck and Tiesberg, 1993..

between greenhouse gas concentrations and global temperature. Endogenous to a model of climate and the economy, we observe realized temperatures over time, possibly perturb emissions to obtain a greater climate signal, and as a result learn the underlying value of climate sensitivity.

In the following sections we detail each of these three approaches to learning and discuss the existing literature as it pertains to climate change.

A. Passive Learning

The typical approach to including passive learning is to posit a two or three period model where uncertainty changes from one period to the next. Miller and Lad (1984) use a two period model with an *ex ante* probability distribution on period i benefits (b_i) of $f(b_1, b_2)$. After observing period one benefits, the *ex post* marginal distribution is obtained: $f(b_1, b_2 * b_1)$. While this is clearly learning, it is often convenient to parameterize the rate of learning so that the effects of the rate of learning can be deduced.

1. Information Structures. Jones and Ostroy (1984), Olson (1990) and Marshak and Miyasawa (1968) provide such a framework through the concept of an ordering on information structures. Start with a set of possible states of nature and a probability vector associated with those states-of-nature being realized. Add to this the receipt of an informative message, and a vector of probabilities of receiving specific messages. The information in the message is a conditional probability on states of nature. An information structure consists of a prior on states-of-nature, a vector of probabilities of receiving specific messages and for each message, an *ex post* probability

of states-of-nature.¹² Of two information structures with the same prior on states of nature, the one that has the greater variability in terms of possible posteriors is viewed as being "more informative." This is equivalent to the more informative structure yielding a higher attainable expected utility when the consumption bundle depends on the state of nature (more flexibility can only be advantageous). Thus if two learning processes are associated with two comparable information structures, then the structure that is more informative corresponds to greater learning.

To quantify this concept of learning further, suppose there is a set of possible states of nature, indexed by $s=1, \dots, S$. Furthermore, suppose there is a finite set, Y , of possible "messages" containing information on the state of nature. Suppose the prior on receiving particular messages is q (dimension equal to the size of Y) and the conditional probability on states of nature (after the message $y \in Y$ has been received) is $p(y)$. We use the term "prior" to refer to a probability distribution on states, before the message is received and posterior to refer to distributions on states-of-nature after a message has been received. Let π be a matrix with columns consisting of $p(y)$ with a different column for each y . Thus π has S rows and the same number of columns as members of Y . (π, q) is an information structure. A first goal is to develop an economically relevant ordering on information structures. A standard definition of the comparative value of information is provided by Jones and Ostroy (1984) (see also Laffont, 1989).

¹²Laffont (1990) refers to this type of information structure as an information structure with noise. An information structure without noise is a special case of one with noise in that messages are mapped into a partition of the set of states of the world; after receiving a message, you know in which subset of the state space the true state of the world lies. Messages that map into a finer partition yield more information.

2. A Special Parameterization of Learning. We consider a special restriction on the set of comparable information structures. In particular, if there are S possible states of nature, we assume a message consists of a noisy signal as to the true state of nature and thus there are S possible noisy signals. Let $\alpha \in [0,1]$ reflect the level of information in the signal with 0 being no information and 1 being perfect information. Thus given a prior \bar{p} we define the star-shaped information structure (α, q) where q/\bar{p} and the s^{th} column of α is

$$p^s = (1 - \alpha)\bar{p} + \alpha e_s \quad (2)$$

where e_s consists of all zeros except for a one in the s^{th} position. Clearly $E_y[p(y)] = \alpha q = \bar{p}$. Furthermore if $\alpha=0$, each column of α is \bar{p} and if $\alpha=1$, $\alpha = I$.

As an example, suppose you can receive one of three messages indicating whether the state of nature is 1, 2 or 3. We are now assuming that number of possible messages equals the number of possible states-of-nature. A message that conveys the maximum amount of information would resolve all uncertainty on the state of nature. If the message is too noisy to contain any information, then the posterior on states of nature is the same as the prior. This is illustrated in Figure 6 where the simplex of probabilities on states of nature is shown. The prior is \bar{p} . The set of posteriors associated with a star-shaped spreading of beliefs, spread all the way out to the vertices, is shown by the three lines radiating out from \bar{p} . Perfect learning would move you to one of the three vertices following receipt of the message. Less learning would move you to one of the three points marked with circles. Even less learning would move you to one of the three points marked with x's after receiving the message.

The advantage of representing learning by this star-shaped spreading of beliefs is that the

process can be parameterized by the θ in equation (4). The disadvantage is that we have eliminated perfectly legitimate and orderable learning processes (emanating from \bar{p} in Figure 1). Kolstad (1993) has incorporated this approach into an integrated assessment model of climate change.

3. Learning and Irreversibilities. There is a modest literature on the effect of passive learning on current period actions when irreversibilities are present. To present these results, we need to set up a simple two period description of the problem.

Consider two time periods, with some action taken in each of these two periods. Thus for $t=1,2$, let U_t and x_t refer to the utility and action taken in time period t . Assume marginal utility is nonnegative and utility is twice differentiable and concave. Furthermore, assume utility in the second period depends on a random parameter, w , taken from a probability space O with a standard measure. After time period 2, some value w will be realized but from the perspective of either time period 1 or 2, there is uncertainty about what that value will be. Between time period 1 and 2, learning takes place and information is gained. We characterize what we have learned by a message y and a posterior distribution on w .

To solve or even specify this problem, we will work backwards from the last time period. Consider time period 2 first, given the message y :

$$\max_{x_2} \int U_2(x_1, x_2, w) \cdot y > \quad (3)$$

$$\text{s.t.} \quad f_1(x_1) \# x_2 \# f_2(x_1) \quad (4)$$

If solutions to (3-4) exist, denote them by the set $X_2(x_1, y)$ which may or may not be a singleton and define $v(x_1, y)$ as

$$v(x_1, y) = \bar{v} [U_2(x_1, X_2(x_1, y), w) \mid y]. \quad (5)$$

The problem in the first period is to find x_1^* that solves

$$\max_{x_1} \bar{v}_y [U_1(x_1) + v(x_1, y)] \quad (6)$$

If a solution to (6) exists, our basic interest is in how x_1^* is affected by different information structures. Define $X_2^*(y) / X_2(x_1^*, y)$. We can summarize current knowledge on the problem in four results.

The first result is due to Simon (1956). He shows that with U_i quadratic and no constraints (in eqn. 4), then the solution, x_1^* , is a function of the mean of w and no higher moments (certainty equivalence applies). This means that without constraint (2), there is no irreversibility effect in the quadratic problem and furthermore, there is no effect of risk aversion on the choice of x_1^* . This is of course due to the significant restrictions on the way in which uncertainty enters utility. Note that rather than restrict U_i and its domain so that a solution is guaranteed to exist, the theorem is confined in its applicability to cases where a solution does exist.

Theil (1957) and then Malinvaud (1969) relaxed these assumptions in developing the concept of first-order certainty equivalence. The functional form restrictions are relaxed at the expense of requiring a much less general representation of uncertainty. Malinvaud (1968) makes

the assumption that the dependence of U_2 on w is small, that constraint (2) is non-binding everywhere, and that the Hessian of U_i is nonsingular in the vicinity of any optimal solution, x_1^* , $X_2^*(y)$. He then shows that x_1^* is independent of uncertainty (first-order certainty equivalence holds).¹³

Both of these results demonstrate that in some cases, uncertain parameters of a problem can be replaced with their expected values without distorting optimal decisions. Malinvaud's (1969) focusing on small amounts of uncertainty suggests that the result is not general; with significant amounts of uncertainty, different distributions of the random parameters may yield different first-period decisions. By extension, different rates of learning may yield different x_1^* .

In both of these results, the constraints imposed by first-period actions on second-period actions had to be ignored (equation 4). This is demonstrated by a counterexample described in Laffont (1980). The constraints (4) characterize the irreversibility effect, first demonstrated by Henry (1974) and Arrow and Fisher (1974). Freixas and Laffont (1984) treat the case where equations (3-4) are of a very specific form, namely that f_1 and U_2 are independent of x_1 and $f_2(x_1) / x_1$ so that eqn 4 becomes $x_2 \neq x_1$. They then prove that if S and S' are information structures with S' more informative than S , if $x_1^*(S')$ and $x_1^*(S)$ exist, and if v in eqn (5) is quasi-concave, then $x_1^*(S') \leq x_1^*(S)$.

This is a statement of the irreversibility effect: if today's actions restrict tomorrow's opportunities (equation 4), then more rapid rates of learning call for a bias in today's actions

¹³This is a loose version of Malinvaud's result. He parameterizes uncertainty and compares the case of no uncertainty with small amounts of uncertainty and demonstrates that x_1^* is constant through a small neighborhood around the certainty point.

towards less restrictions. In environmental matters, if today's actions result in irreversible environmental damage¹⁴, and one is acquiring information over time, then it is optimal to bias today's actions away from causing environmental damage.

In fact the Arrow and Fisher (1974) paper is a special case of this theorem. In that paper they discuss the problem of developing a unit of land vs. preserving the land. If x_i is the amount of land preserved in period i , the amount developed is $1-x_i$. The irreversibility is that once land is developed, it cannot be undeveloped. Thus in a two period world, $x_1 \geq x_2$. A direct application of theorem 3 is that when information is being acquired, x_1 should be larger (i.e., bias the decision in favor of the environment). This is the Arrow and Fisher (1974) result. A further implication is that the faster one is learning, the more land should be preserved in period 1. Thus the bias goes up with the rate of learning.

The most general result is due to Epstein (1980). He shows that depending on the shape of the expected value of v in eqn. (5), then learning may cause current period actions to be higher or lower than with slower (or no) learning.

While his result is quite general, it does not give very direct and transparent results. A number of authors have developed more definitive results for specific forms of the utility functions in eqn (3). Epstein (1980) himself presents several examples as do Hanemann (1989), Ulph and Ulph (1995) and Kolstad (1996).

B. Active Learning

¹⁴Irreversibility is not a biologic or physical term. Obviously, once you cut down a tree, you cannot re-root it, at least not immediately. However, if you are cutting down ten trees a year, cutting one more in one year is not irreversible since it can be corrected the following year.

The most common type of learning in the economics literature is termed here active learning. The experimental process of information generation is endogenous to the model, although in most cases that process is very simple. An agent is able to manipulate variables under her control in such a way as to generate information. For instance, if a monopolist is unsure about the demand function she faces, she may perturb price, observe quantity sold and learn about the underlying structure of demand. This is the example considered by Rothschild (1974) and more recently by such authors as Balvers and Cosimano (1990). One result of this work is that a firm will always end up charging a particular price, but it may not be the "right" price. The firm may stop learning before uncertainty has been completely resolved and thus may operate using an incorrect price. This leads to price dispersion. Easley and Kiefer (1988) generalize this result, showing that when one is controlling a stochastic process with unknown parameters, there may never be convergence to true parameter values; in fact, there may be convergence to a variety of parameter values.

Grossman et al (1977) consider an example closer to that of climate change. They consider the problem of a drug of unknown efficacy. By taking the drug and observing health, the user learns about the relationship between drug consumption and health. Consider the analogous problem from climate change. Most climatic processes are stochastic. In particular, the average annual global temperature is well recognized to be stochastic, with some deterministic elements, such as radiative forcing from increased levels of greenhouse gases. Consider the simplest representation of this process:

$$T_{t+1} = \beta \ln M_t + e_t \tag{8}$$

where T_t and M_t are temperature and greenhouse-gas concentrations (relative to some base) at time t , β is a constant, and e_t is a random shock, assumed to have a zero mean but perhaps exhibiting serial correlation.

Suppose we do not know β . How can β be learned? The way to learn about β is to try widely differing values of M , observing the realized temperature. For instance, suppose β can take one of two values: β_H and β_L . Suppose further, that we have a prior of p on β_H . Then p will optimally evolve according to Bayes rule:

$$p_{t+1} = \frac{f(T_{t+1} | \beta_H) p_{t+1} + f(T_{t+1} | \beta_L) p_{t+1}}{f(T_{t+1} | \beta_H) p_{t+1} + f(T_{t+1} | \beta_L) p_{t+1}} \quad (8)$$

Figure 7 shows how a prior on p might evolve for typical parameter values in eqn. (7) (taken from Kelly and Kolstad, 1996). Note that the greater the value of M , the easier it is to resolve the value of β . Basically, larger values of M give greater signal to noise ratios, permitting quick learning. Note that with a value of M just 10% above historic levels, it can take several decades to resolve the value of β .

Clearly, one learns most rapidly by having large values of M ; however that may also be costly because of the disutility of M . Thus there is a tension between learning more rapidly and reducing the disutility of M . In the Grossman et al (1977) paper, the issue was the cost of the drug vs. the information gain from using it.

Active learning can be included in an integrated assessment model of climate change. Referring back to eqn. (1) which is our generic integrated assessment model. If one of the states

of the problem is the prior on an uncertain parameter (α or β), one can specify the manner in which the state evolves, according to Bayes Rule (eg, eqn. 8). This becomes one of the state transition equations in eqn. (1b). The model is then a stochastic dynamic program. It can be solved using a stochastic version of the Bellman equation. This is done in the context of an integrated assessment model of climate change in Kelly and Kolstad (1996). In that paper, uncertainty in climate sensitivity is the issue. One of the conclusions of the paper is that it can take five to fifteen decades to resolve uncertainty regarding climate sensitivity, given the assumptions used in the model.

C. Purchased Information

In the previous section, the learning process was made explicit; the control variable generated information and short-term utility. The tension is between using the control to obtain the maximum amount of information and using the control to obtain the maximum amount of utility. The industrial organization literature is somewhat more explicit about the cost of obtaining information. In the research and development literature,¹⁵ the cost side of the equation is more akin to a production function where the precise mechanism of information generation is hidden within the production process.

A typical approach is given in Grossman and Shapiro (1986). In their model, there is a payoff, W , associated with accumulating a quantity of knowledge, L , which may be random. In each period, the agent can purchase a certain amount of information, l_t , at cost $c(l_t)$. When $j = t$,

¹⁵See Reinganum (1989) for a review of related literature.

\$ L, the payoff is realized. The issue is the time profile of investments in R&D and the time to success. These models are usually found in the literature on patents and the typical issue is the effect of competition or monopoly on innovation. The literature is not usually concerned with the evolution of a prior on some random variable or state of the world.

Perhaps a better analog is that of search theory. Rothschild (1974), in a classic paper, presents a model whereby consumers learn about prices by searching for them. Consumers start with a diffuse prior on prices and then each period "draw" a price at a cost. They then update their prior on the distribution of prices. The problem is one of optimal stopping: when to stop sampling prices and be satisfied with the resulting distribution of prices.

This process of information acquisition could be endogenized into an integrated assessment model of climate change although it might be quite difficult to determine the appropriate cost for a draw from the distribution of climate parameters.

IV. CONCLUSIONS

This paper has been motivated by the need to include issues of risk, uncertainty, irreversibility, learning and catastrophe in analyses of climate change, particularly integrated assessment models used to investigate climate change policy. Our approach to this problem has been two-fold. First we have explored these issues in general in the context of climate change. Secondly, we have investigated uncertainty and learning have been dealt with in economics, with a particular eye on how learning can be included within equilibrium models such as those used to analyze climate change--integrated assessment models.

Our conclusion is that this is a fertile research area in which some progress has been made

but in which much remains. In particular, treating adaptation as a learning process would seem to offer the greatest potential for adding insight and structure to the economics of climate change.

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